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A FORTRAN PROGRAM FOR THE UPWARD CONTINUATION OF GRAVITY ANOMALIES

by

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FOREWORD

This report was prepared by Dr. Richard H. Rapp, Assistant Professor, Department of Geodetic Science of The Ohio State University under Air Force Contract No. AF19(628)-5701, OSURF Project No. 2122. The contract covering the research is administered by the Air Force Cambridge Research Laboratories, Office of Aerospace Research, Lawrence G. Hanscom Field, Bedford, Massachusetts, with Mr. Owen Williams and Mr. Bela Szabo, Project Scientists.

The original upward continuation program was written by Doug Fleckner in the programming language SCATRAN used at the Ohio State University Computer Center. This program was then translated to the FORTRAN II version by Tom Tripp, also of the University Computer Center.

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ABSTRACT

The equations of Moritz have been utilized in a computer program for the upward continuation of gravity anomalies. The solution for the anomaly at some elevation H is obtained through the evaluation of the Poisson integral using the known anomalies in the surrounding areas on the ground. The input to the program consists of blocks of 5'x 5' mean free-air anomalies, as well as 2!5x 2!5 mean free-air anomalies for elevations below 12 km. The upward continued anomaly is thus computed for a sequence of points at, up to, 15 different input elevations. The lowest elevation that can be used is on the order of 4 km, while the highest elevation is limited by economical reasons to an elevation on the order of 50 km. In addition to the program itself numerous tests are described that pertain to square size and elevation limits to be used in the program.

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A FORTRAN PROGRAM FOR THE UPWARD CONTINUATION OF GRAVITY ANOMALIES

1. Introduction

The upward continuation of gravity anomalies may be defined as the computation of the gravity anomaly at some elevation H, given the gravity anomalies on the surface of the geoid, H = 0.

This problem has been examined by several authors in attempts to formulate theoretical solutions and to make practical computations. Two solutions are generally formulated: the first utilizes some form of a template; the second uses anomalies estimated in blocks of various sizes.

Template method investigations have been concerned especially with finding suitable compartment sizes or suitable points at which the gravity anomalies should be read on some grid surrounding the subpoint of the computation stations. Particular reference should be made to works of Henderson, Hirvonen, Peters, Tsuboi and others. The template method has the advantage that it maintains its usefulness for very low elevations, say on the order of 1 km or lower, because small compartments are utilized around the computation point. By using compartments at large distances from the computation point it is possible to extend the upward continued anomalies to the higher elevations, on the order of hundreds of kms. The disadvantage of the template method is that a point to point estimation of the mean anomalies in the various compartments is necessary. Thus considerable time and effort will be expended using the template method which should be critical for a production type evaluation of the upward continued anomaly. The template method is fundamentally manual and little utilizations may be made of a digital computer in obtaining the results.

The second method of upward continuation involves the use of mean anomalies in pre-determined sizes. These blocks are generally bordered by latitude and longitude grid lines. It is thus possible to estimate the mean values with no need for re-estimations when the computation point shifts. This is a distinct advantage over the template method. However, there is some question on the size of the blocks. Mean anomalies may be usually found in $5^\circ \times 5^\circ$, $1^\circ \times 1^\circ$ and $5' \times 5'$ blocks. These sizes are perhaps sufficient for the higher elevations but smaller size blocks are needed for lower elevations. This will require additional estimations near the computation point, but which in any event will be less, in most cases, than that required by the template method. Additional discussions concerning the block sizes needed will be found later in this report.

We thus choose the method of upward continuation which utilizes squares (or blocks) of various sizes. We shall limit the discussion and the computer program to those elevations (on the order of 200 kms) at which the earth may be approximated by a plane.

The theory and equation to be used in this report are not new. We will freely draw on the works of Hirvonen and Moritz in the discussion of this problem.

2. The Theory

2.1 Upward Continuation

The fundamental solution to the problem is found through the Poisson Integral:

$$V_p = \frac{r^2 - R^2}{4\pi R} \iint \frac{V_s}{D_s^3} \cdot d\sigma \tag{2.1}$$

V_p: potential function at some point external to the sphere

r : distance from center of sphere to the point

R : radius of sphere

V_s :function on the surface of the sphere (this function does not need to be a harmonic function)

D_s: distance from the surface element (V_s) to P

$d\sigma$: elemental area on the sphere

H : elevation above the sphere ($r = R + H$)

In terms of the gravity anomalies we have:

$V_p = r\Delta g_H$

$V_s = R\Delta g$ where Δg_H is the anomaly at elevation H and Δg is the anomaly on the surface. Then (2.1) becomes:

$$r\,\Delta g_H = \frac{r^2 - R^2}{4\pi R} \iint \frac{R\,\Delta g}{D_s^3}\, d\sigma$$

or

$$\Delta g_H = \frac{r^2 - R^2}{4\pi R} \iint \frac{R\,\Delta g}{D_s^3}\, d\sigma \tag{2.2}$$

Equation (2.2) is usually transformed into another form by the substitution:

$$t = \frac{R}{r} = \frac{R}{R + H} \quad .$$

Then:

$$D_s^2 = R^2 + r^2 - 2Rr \cos \psi$$

$$D_s^2 = r^2(1 + t^2 - 2t \cos \psi)$$

so

$D_s = rD$ where $D = (1 + t^2 - 2t \cos \psi)^{\frac{1}{2}}$ with ψ, the spherical distance between the surface element and the computation point. In addition if we let $d\sigma_1$ be the area on a unit sphere:

$$d\sigma = R^2 \, d\sigma_1$$

we have:

$$\Delta g_H = \frac{R^2(r^2 - R^2)}{4\pi r^4} \iint \frac{\Delta g}{D^3} \, d\sigma_1$$

or

$$\Delta g_H = \frac{\frac{R^2}{r^2}\left(1 - \frac{R^2}{r^2}\right)}{4\pi} \iint \frac{\Delta g}{D^3} \, d\sigma_1$$

or using t:

$$\Delta g_H = \frac{t^2(1 - t^2)}{4\pi} \iint \frac{\Delta g}{D^3} \, d\sigma_1 \tag{2.3}$$

Equation (2.3) corresponds to equation (8.2) of Moritz [3] and is called by him the upward continuation integral. This equation may be used for the upward continuation of gravity anomalies.

There is, however, another form that may be obtained from (2.3) which is valuable in examining the case of plane assumptions. Using the substitution:

$$t = \frac{R}{R + H}$$

$$t^2 = \frac{R^2}{(R + H)^2}$$

$$1 - t^2 = \frac{(R + H)^2 - R^2}{(R + H)^2} = \frac{2H\ (R + H/2)}{(R + H)^2}$$

$$D^3 = \frac{D_s^3}{(R + H)^3}$$

we have:

$$\Delta g_H = \frac{R^2\ 2H\ (R + H/2)\ (R + H)^3}{4\pi\ (R + H)^2\ (R + H)^2} \iint \frac{\Delta g}{D_s^3}\ d\sigma_1$$

$$\Delta g_H = \frac{H}{2\pi}\ \frac{R + H/2}{R + H} \iint \frac{\Delta g}{D_s^3}\ d\sigma$$

Now the ratio $\frac{R + \frac{H}{2}}{R + H}$ is close to 1 : $\simeq 1 - \frac{H}{2R}$.

For the small elevations envisioned in these computations we will take this ratio to be one. At H = 200 km, the ratio is .985. We thus write the second form of the upward continued equation as:

$$\Delta g_H = \frac{H}{2\pi} \iint \frac{\Delta g}{D_s^3}\ d\sigma \tag{2.4}$$

Equation (2.4) can also be transformed into a form suitable for use on a plane. These may be done by associating the distance D_s with a distance D on a plane and the spherical area element $d\sigma$ with a plane area element dx dy. If we thus set up a rectangular coordinate system with origins at the subcomputation point, x axis positive east and y axis positive north we can write:

$$\Delta g_H = \frac{H}{2\pi} \iint \frac{\Delta g}{D_o^3} \, dx \, dy \tag{2.5}$$

where $D_o^2 = x^2 + Y^2 + H^2$.
Equation (2.5) is the form of the upward continued integral in a plane approximation. Errors associated with it in comparison to the true equation limit its use. According to Moritz [3, p.71] this limit is for a distance up to 20° from the computation point for a height less than 250 km.

The computation of the x, y coordinate needs now to be discussed. The basic problem involves the transformation of some latitude, longitude coordinate difference on a sphere to a corresponding x, y coordinate on a plane. The obvious method would be some sort of map projection that would have minimum amounts of scale error and angular distortions within a range of the computation point. It is not necessary however to search for this because only simple approximations and equations may be used. For example Moritz suggests:

$$\begin{cases} x = R \cos\varphi \, (\lambda - \lambda_p) \\ y = R \, (\varphi - \varphi_p) \end{cases} \tag{2.6}$$

where φ and λ are the coordinates of the particular surface point of interest and φ_p and λ_p are the coordinates of the actual computation point.

As the evaluation of (2.5) must be done as a finite summation we write:

$$\Delta g_H = \frac{1}{2\pi} \sum \Delta g \;\; c(\varphi,\lambda) \tag{2.7}$$

where the summation is to be taken over all the used anomalies and c (φ,λ) has two forms. The first is a direct form that is easily found from (2.5). The second is an integrated form. The latter is necessary for small elevations near the computation point.

The first form is:

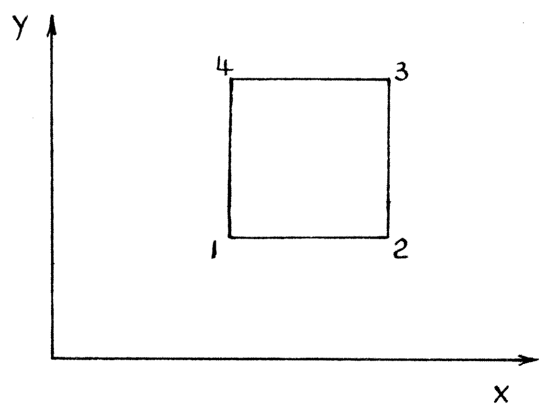
$$c\;(\varphi,\lambda) = \frac{H}{D_o^3} \;\; \Delta A \tag{2.8}$$

where ΔA is the area of the elemental surface block. φ and λ are taken as the coordinates of the center (mean Coordinates) of the block under consideration. It can be seen that this form is satisfactory for more distant blocks but it will be sensitive to the latitude and longitude evaluations as the square is closer to the computation point. To this end Moritz computes an integrated form of c (φ,λ). This is:

$$\bar{c}\ (\varphi,\lambda) = \frac{1}{\Delta A} \iint c\ (\varphi,\lambda)\ dA$$

$$= \iint \frac{H\ dA}{D_o^3}$$

where the integration is carried over the rectangle shown below;



For this square we have:

$$\bar{c}\ (\varphi,\lambda) = \bar{c}\ (\varphi,\lambda) = \iint \frac{H\ \ dx\ dy}{(H^2 + x^2 + y^2)^{3/2}}$$

Moritz gives the solution in the form:

(2.9) $$\bar{c} = F\ (1) - F\ (2) + F\ (3) - F\ (4)$$

where:

(2.10) $$F\ (x,y) = \tan^{-1} \frac{xy}{HD_o}$$

In (2.9) the value F (i) indicates that these evaluations are to be done using (2.10) with the x, y coordinates of point i as indicated in the figure. The latitude used in the evaluation of the x coordinate (i.e. in cos φ) must be the same for point (1,2) and (3,4). This will then assure that the figure described by the coordinates is a rectangle as required by the derivation for $\bar{c}$. In fact this latitude is not critical and several variations are possible; for example, the northern latitude, the southern latitude or the mean latitude of the square in question. In the program which will be developed in this report, the northern latitude of the square is used.

The above completes the general discussion of the theory involved with the upward continuation of gravity anomalies.

2.2 Normal Gravity

Besides the interest in upward continuation of gravity anomalies, there is also some need for the normal gravity at the elevation, latitude, and longitude at which the upward continuation is to take place. The normal gravity can simply be defined as the sum of the attraction and centrifugal force of a point rigidly attached to a rotating ellipsoid of revolution whose surface is an equipotential surface.

Several authors have discussed this problem. A previous report [6] in this series has explained and summarized a solution due to Hirvonen. In that report the normal gravitation and normal gravity along several coordinate systems were computed. The specific need in this report is for only normal gravity in a direction perpendicular to the given equipotential surface of the reference ellipsoid. This direction is normally associated with the direction of the geodetic latitude, φ.

Since extensive discussion has taken place about the various methods, none will attempted here. However, since the normal gravity in the φ direction is given in the computer program to follow, the specific equations to be used for this vertical component of normal gravity will be given in a later section.

3. Equations

3.1 Upward Continuation

The fundamental equation is:

$$\Delta g_H = \frac{H}{2\pi} \iint \frac{\Delta g}{D_o^3} \, dx \, dy$$

or in a summation form,

$$\Delta g_H = \frac{1}{2\pi} \sum \Delta g \; c\,(x,y)$$

where the summation is carried out for each block containing a mean anomaly. In this program we will use 5'x 5' blocks and 2.'5 x 2.'5 blocks, so that the summation must be carried over each type of square. In addition the direct or non-integral form of $c(x, y)$ or $c(\varphi,\lambda)$ as given in (2.8) will be used for the 5' squares. In this case the area of the 5' square would be computed as:

$$\Delta A = (R\,\Delta\varphi\; R\cos\varphi\Delta\lambda)$$

$$= \cos\varphi \left(\frac{R \cdot 5'}{3437.7468}\right)^2$$

$\Delta A = 85.86328 \cos\varphi$ where $R = 6371$ km.

For the smaller 2.'5x 2.'5 blocks that will be used close to the computation point the integrated form as given in (2.9) and (2.10) will be used. Further discussions will be found in a subsequent section of this report concerning these square sizes.

Given the following information:

φ_o, λ_o, H_1, H_2 --- H_k of the computation point

one set of 5'x 5' mean free-air anomalies $\Delta g\ (i,j)$
(center point coordinates φ_i, λ_i)

one set of 2.'5x 2.'5 mean free-air anomalies $\Delta g\ (i,j)$

Compute:

Δg_{H_1}, Δg_{H_2}, --- Δg_{H_k}

A. Sum over all 5'x 5' blocks (i,j):

For each block (i,j):

(3.1) $$x = 111.19493 \cos \varphi_i \; (\lambda_i - \lambda_o)$$

(3.2) $$y = 111.19493 \; (\varphi_i - \varphi_o)$$

For each elevation (k), for each block:

(3.3) $$D^2_{i,j,k} = x^2 + y^2 \quad H^2_k$$

(3.4) $$\frac{c_{i,j,k}}{\Delta A} = \frac{\cos \varphi_i \; H_k}{D^3_{i,j,k}}$$

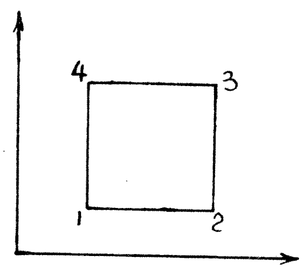
(3.5) $$\Delta g_{h_k} = \Delta g_{H_k} + \Delta g \; (i,j) \cdot c_{i,j,k}/\Delta A$$

(3.6) $$\Delta g_{H_k} = \Delta g_{H_k} \;\; \Delta A$$

B. Sum over all 2.'5x 2.'5 blocks (i,j)

For each block (i,j):

For each corner of the block:



(3.7) $$x_1 = 111.19493 \cos \varphi_4 (\lambda_4 - \lambda_o)$$

(3.8) $$y_1 = 111.19493 \; (\varphi_1 - \varphi_o)$$

(3.9) $$x_2 = 111.19493 \cos \varphi_4 \; (\lambda_2 - \lambda_o)$$

(3.10) $$y_2 = y_1$$

(3.11) $$x_3 = x_2$$

(3.12) $$y_3 = 111.19493 \; (\varphi_3 - \varphi_o)$$

(3.13) $$x_4 = x_1$$

(3.14) $$y_4 = y_3$$

For each elevation (k), for each block (i,j)

$$D = \sqrt{x^2 + y^2 + H_k^2} \tag{3.15}$$

$$c_{i,j} = F(1) - F(2) + F(3) - F(4) \tag{3.16}$$

$$\text{where } F = \tan^{-1} \frac{xy}{HD} \tag{3.17}$$

$$\Delta g_{H_k} = \Delta g_{H_k} + c_{i,j}\, \Delta g_{i,j} \tag{3.18}$$

The above complete the fundamental equations used for the upward continuation of anomalies.

3.2 Normal Gravity

The equationsused for the computation of the vertical component of normal gravity are below. with the φ, λ, and H of the computation point

	compute	find
(3.19)	$X = (N + H) \cos\varphi \cos\lambda$	X
(3.20)	$Y = (N + H) \cos\varphi \sin\lambda$	Y
(3.21)	$Z = (N(1 - e^2) + H) \sin\varphi$	Z
(3.22)	with $N = \dfrac{a}{(1 - e^2 \sin^2\varphi)^{\frac{1}{2}}}$	
(3.23)	$p^2 = x^2 + y^2$	p^2
(3.24)	$r^2 = p^2 + Z^2$	r^2
(3.25)	$K^2 = r^2 + c^2$	K^2
(3.26)	$h^2 = \sqrt{K^4 - 4p^2c^2}$	h^2
(3.27)	$\sin^2\alpha = \dfrac{K^2 - h^2}{2p^2}$	α
(3.28)	$\tan\beta = \dfrac{Z}{p\cos\alpha}$	β

compute		find
(3.29)	$q = \frac{1}{2}[\alpha - 3\cot\alpha\,(1 - \alpha\cot\alpha)]$	q
(3.30)	$q' = \dfrac{3(1 - \alpha\cot\alpha)}{\sin^2\alpha} - 1$	q'
(3.31)	$w = (1 - \sin^2\alpha\cos^2\beta)^{\frac{1}{2}}$	w
(3.32)	$\Gamma_\alpha = \dfrac{KM\sin^2\alpha}{c^2 w} + \dfrac{\omega^2 a^2 q'\sin^2\alpha}{2cq_o w}\left(\sin^2\beta - \dfrac{1}{3}\right)$	Γ_α
(3.33)	$\Gamma_\beta = \dfrac{\omega^2 a^2 q\sin\alpha\sin\beta\cos\beta}{cq_o w}$	Γ_β
(3.34)	$\gamma_\alpha = \Gamma_\alpha - \dfrac{\omega^2 c\cos^2\beta}{w\tan\alpha}$	γ_α
(3.35)	$\gamma_\beta = -\Gamma_\beta + \dfrac{\omega^2 c\sin\beta\cos\beta}{w\sin\alpha}$	γ_β
(3.36)	$\gamma_T = \sqrt{\gamma^2_\alpha + \gamma^2_\beta}$	γ_T

where γ_T is the normal gravity in the vertical direction. Certain constants used in this program designated by c, KM, q_o, a, e^2 refer to the International Ellipsoid and may be computed as described in the pertinent report.

4. Special Considerations

In the previous section we have tacitly assumed several procedures which are now appropriately discussed. The first question to be examined is why use 5'x 5' blocks, and why use 2'.5x 2'.5 blocks. Secondly, why use the integrated form of $c_{i,j}$ only for the 2'.5x 2'.5 blocks, if at all.

4.1 Consideration of Block Size

The first question may be answered by considering two factors. The first is the area over which the integral summation is to be carried. In theory, the summation should be made over the entire sphere, or in the plane approximation over the entire plane. However, the effects of the distant area are small for the elevations (< 200km) of interest in this report. Moritz has estimated that it is sufficient to extend the summation over a radius equal to 10 times the height of the point. For a height of 10km, the radius would be 100km or approximately 1° in latitude. Areas of this size are most readily described in terms of 5'x 5' mean anomalies. For the higher elevations, the area to be considered would be larger and perhaps more easily represented by large size elements. However, this would complicate data preparations. It is thus felt that for elevations that would be considered for airborne measurements and similar encountered altitudes, 5'x 5' mean anomalies are sufficient. For the higher altitudes under consideration (100-200km), this size block is perhaps too small for the most economical computation, but they will satisfy the requirements.

The lowest elevation at which a computation may be made is limited by the size of the squares used in the integration. As the value of H becomes smaller, the size of the blocks in which in which the mean anomaly has been estimated must also be reduced in order to be able to adequately reflect the variations of the anomaly field. If the elevation was only $dH > 0$, the needed squares would be essentially point anomalies.

It is pertinent to discuss the elevation at which a certain size square will give satisfactory results. Various authors have made suggestions along these lines, either explicitly or implictly. For example[2] Hirvonen has computed a template for the upward continuation. He concluded from his figures that 5'x 5' squares would not give the same accuracy as the template below 20km. On the other hand, the smallest radius of his template has a radius of 0.4H, so that the radius for 20km = 8km. Now a circle of 8km would be composed of approximately 4 5'x 5' squares, which if the $c_{i,j}$ are calculated in the integrated form, should yield better accuracy than the template. If we use the criteria of 0.4H the lowest elevation corresponding to the accuracy of the template method would be:

$$H = \frac{a}{2(.4)} = \frac{a}{.8}$$

where a is the side of the square. Taking for a 5'x 5' block, we have a = 8km so that H = 10km. In words we might conclude that the size of the square needed is approximately 0.8 of the point elevation.

Moritz [3] recommends another procedure when dealing with square elements. This involves evaluating the upward continued anomalies at the center of adjacent 5'x 5' squares surrounding the center square in which the computation point lies. Interpolation for the given point is then carried out from the 3 given values. This procedure is said to be necessary for elevations smaller than 70 km. Even though an interpolation is performed in this method, it still should not readily improve the situation for the lower elevation (~10 km).

In addition to the above described method, Moritz also described a method in which the effects of gradients within a 5'x 5' square may be taken into account for the non-vertical components for elevations less than 10km.

Orlin has also mentioned this problem of square size. He adopts a figure giving the ratio of distance to block size as 2 to 1. Thus for an elevation of 16 km a square size of 8 km (5') would be needed. However, these figures were only used to within 10' of the computation point, with a template procedure being applied for that inner zone.

We might formulate the problem in a slightly different way. Moritz [4] has given an expression for the expected accuracy of the upward continued anomaly based on a rectangular block with side a and b containing the mean anomaly. This is:

$$m_H^2 = \frac{1}{H^2} \frac{A}{8\pi} m^2$$

where A is the area (ab) of the block and m^2 is the expected standard error of the estimated mean anomaly in the square. m^2 may be taken as:

$$m^2 = \frac{\alpha^2}{6} (a^2 + b^2)$$

where α^2 is a constant depending on the covariances of the anomalies. Using this we can write:

$$m_H^2 = \frac{\alpha^2}{48\pi} \frac{(a^2 + b^2)\ ab}{H^2}$$

Now suppose that we ask the following question:

What is the size of a block (a_2, b_2) needed at a certain elevation H_2, to yield the same accuracy at an elevation H_1 with a block size a_1, b_1?

We answer this by taking the ratio of the respective standard errors:

$$\frac{m_{H_2}^2}{m_{H_1}^2} = \frac{H_1^2}{H_2^2} \frac{(a_2^2 + b_2^2)\ a_2 b_2}{(a_1^2 + b_1^2)\ a_1 b_1}$$

If we approximate the rectangular blocks with squares, $a = b$, we have:

$$\frac{m_{H_2}^2}{m_{H_1}^2} = \frac{H_1^2}{H_2^2} \frac{a_2^4}{a_1^4}$$

If the accuracies at these elevations are to be the same, this ratio must be 1, so we have

$$a_2 = a_1 \sqrt{\frac{H_2}{H_1}}$$

Let us accept the value of a_1 as 8km (5' sq) and compute the value of a_2 corresponding to selected values of H_1 and H_2. These results are presented in Table 1.

Table 1

Value of a_2 for given H_1 and H_2 (km)

H_2 \ H_1	20	15	10	5
15	6.9	—	—	—
10	5.7	6.5	—	—
5	4.0	4.6	5.7	—
4	3.6	4.1	5.1	7.2

If we accept adequate accuracy from 5' square at H = 20km, we would expect a similar accuracy at 5km with a 4km (2!5) square. Other conclusions follow similarly.

It is apparent from the figures that there is some disagreement on what size of block should be used for the lower elevations. On the other hand, whether we use the rule of 0.8H for a_2 or the a_2 from Table 1 it appears that the use of 2!5 squares is valid for elevations down to 5km and perhaps 4km. Tests pertinent to these conclusions are described in the next section.

4.2 Integration Considerations

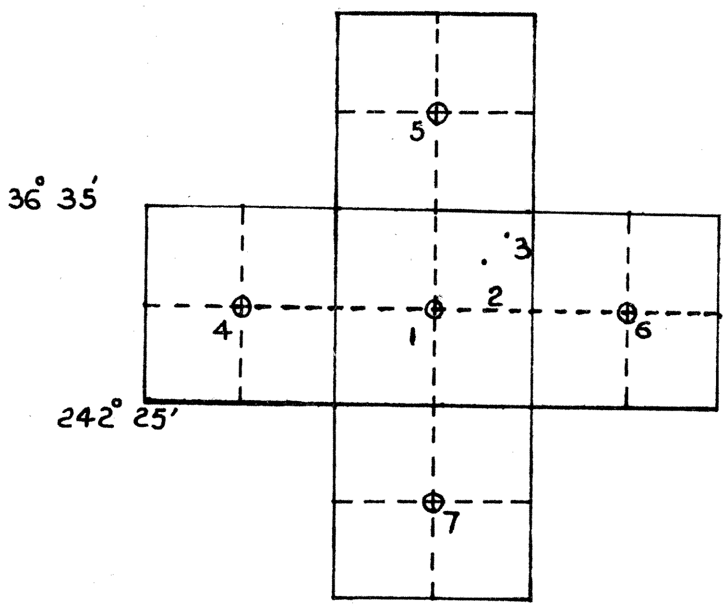
We have discussed two forms of $c_{i,j}$, a non-integrated form and the integrated form. The question arises where and with what size squares should the particular forms be used. Granting that it is sufficient to use 5'x 5' and 2!5x 2!5 blocks, is it sufficient to use the integrated form of $c_{i,j}$ for the 2!5 blocks, and the non-integrated form for the 5'x 5' blocks?

To answer the above question a test situation was examined under various conditions. A 3° (φ) x 5° (λ) area was selected with the coordinates:

$38^\circ \rightarrow 35^\circ$

$240^\circ \rightarrow 245^\circ$ (115° - 120°)

A set of 5'x 5' mean free-air anomalies was established for this area from estimated Bouguer anomalies and mean elevations. Near the center of this area a group of stations was selected for these test computations. The location of these points is described below:



Though points 1, 2, and 3 are of primary interest, the actual coordinates for all the points are now given:

Point	Latitude	Longitude
1	36° 32' 30''	242° 32' 30'', -117° 27' 30'' W
2	36 33 45	242 33 45, -117 26 15 W
3	36 34 00	242 34 00, -117 26 00 W
4	36 32 30	242 27 30, -117 32 30 W
5	36 37 30	242 32 30, -117 27 30 W
6	36 32 30	242 37 30, -117 22 30 W
7	36 27 30	242 32 30, -117 27 30 W

Points 1, 4, 5, 6 and 7 lie at the center of 5'x 5' squares. Point 2 is located at the center of a 2!5x 2!5square while point 3 has no special location, except lying within that same 5' block as points 1 and 2.

Points 5 and 6 are needed for an interpolation (with point 1) for finding Δg_H from the so-called sub-station. The method for interpolation is described by Moritz and basically is the fitting of a plane through points 1, 5 and 6 which lie at 5'x 5' centers and interpolating for the gravity anomaly at points 2 and 3. If we consider a coordinate system with the origins at point one and computing $x = 6371 \cos \varphi_o (\lambda - \lambda_o)$ and $y = 6371 (\varphi - \varphi_o)$ where φ_o, λ_o are the coordinates of point one, we can form an expression for the Δg_H using equation 5 of Moritz [5]. We find:

for point 2

$$\Delta g_{H2} = \frac{\Delta g_1}{2} + \frac{\Delta g_5 + \Delta g_6}{4}$$

for point 3

$$\Delta g_{H3} = .40\ \Delta g_1 + (\Delta g_5 + \Delta g_6) \cdot .3$$

The anomalies around the three points of interest (1,2,3) were chosen in three different forms:

1. the original 5'x 5' mean free-air anomalies
2. the 2!5x 2!5 mean anomalies with the same mean in this square as in the corresponding 5' square
3. the 2!5x 2!5 mean anomalies with various values in them, computed such that their mean would be the same in the corresponding 5'x 5' square.

The situation as described in the best form is shown in Figure I.

Figure I

5'x 5' Means for Test

36° 40'

23.5	42.4	1.1
71.3	72.6	36.3
2.4	-34.9	35.5

36° 35' — 36° 30' — 36° 25'

242° 25' — 30' — 35' — 242 40'

The situation as described in the second form is identical with that of Figure I except within each square are four values, the same values as in Figure I representing the 2!5 mean anomalies.

The situation as described in the third form is shown in Figure II

Figure II

36° 40'

-16.0	20.0	39.6	40.0	10.0	0.0
40.0	50.0	60.0	30.0	10.3	4.1
96.2	73.0	74.0	85.0	65.0	30.0
47.0	69.0	91.4	40.0	55.2	-5.0
13.2	26.2	-42.1	10.0	61.5	24.7
-4.7	-25.1	-40.1	-67.4	34.1	21.7

36° 35' — 36° 30' — 36° 25'

242° 25' — 30' — 35' — 242 40'

2!5x 2!5 Mean Anomalies in Test Case

The values of Δg_H were computed at 10 elevations using 6 different methods or anomalies. These methods are now described:

A - using only 5'x 5' means, c_{ij} not integrated

B - using only 5'x 5' means, c_{ij} integrated

C - using only 5'x 5' means, c_{ij} integrated, values interpolated from substation calculation using surface fitting

D - using 5'x 5' means, c_{ij} integrated and with 2!5x 2!5 means with c_{ij} integrated, with the same mean in the 2!5 square as in the corresponding 5' square.

E - using 5'x 5' means with c_{ij} non-integrated, and with 2!5x 2!5 means with c_{ij} integrated, with the same mean in the 2!5 square as in the corresponding 5' square.

F - using 5'x 5' means with c_{ij} non-integrated, and with 2!5x 2!5 means with c_{ij} integrated, with various mean values (Fig. II) but with the same overall mean as the 5' square.

These results for these computations are presented in Table 1 where the evaluations at points 1, 2, and 3 were made. There are several conclusions that may be drawn from this table. These will be drawn for the lowest elevation as the differences between the methods decreases with elevation, as does the anomaly itself.

For cases A through E, the method that should be regarded as best would be case D. Method B and Method D should give identical results since the same mean anomalies have been used in the two square sizes. It is apparent from Table 1 that Method A gives incorrect results when the point is at the center of the 5' square and fair results otherwise. The smallest difference between the two methods is .4 mgals at the highest elevation while the elevation at which the deviation does not exceed 1 mgal is near 12 km. Method C yields results consistently lower than the accepted value found in Method D. At points 2 and 3 an error of 4 mgals is found for the lowest elevation, while the result at 16 km shows an error of 0.5 mgal. Even greater discrepancies are apparent when compared with the results from Method F, but the real comparison of interest is with Method D. It appears that this method of interpolation yields erroneous results in this example and there is no reason to believe it would not hold for other situations. Besides this fact, additional work is required for the computation of the anomalies at the substations which can substantially increase the needed computer time.

Method E enables us to see if it is possible to use only the integrated c_{ij} for the $2\frac{1}{2}$x $2\frac{1}{2}$ means while using the direct form for the 5' squares. The answer appears to be yes, as the maximum difference between E and D was 0.1 mgals.

It is of interest to compare Method F, where rather erratic anomalies for the 2!5 blocks have been used, with Method D. If these anomalies had actually existed as error of using D over F would be 2.4, 3.5, and 3.8 mgals. for points 1, 2, and 3 respectively at H = 4 km. The elevation at which the difference between D and F is near 1 mgal is at 12 km for the three points. We could than say that below 12 km we should use smaller size squares if an accuracy of integration below 1 mgal is to be maintained. From Table 1 (or the equation for a_2) we could than conclude that the 2!5x 2!5 blocks may be used down to an elevation of 4 km.

In summary, we may tentatively reach the following conclusions:

1. It is necessary to use the integrated form of c_{ij} only for the closest squares which are the 2!5x 2!5 blocks, with the un-integrated form of c_{ij} being used for the farther 5'x 5' blocks.

2. If no 2!5x 2!5 blocks are used, the non-integrated form of c_{ij} for the 5' means should be used no lower than 12 km.

3. If no 2!5x 2!5 blocks are used the integrated form of c_{ij} for the 5' means should be used no lower than 12 km.

4. The substations computation and subsequent surface interpolation does not give correct results for the lower elevations.

Points 2 and 3 need comment as taken together they look trivial. However, the reasoning behind each is different. Statement 2 arises from the inaccuracy in the non-integrated c_{ij} over the c_{ij} in the integrated form. Statement 3 arises from an expected variation in the anomaly field in the surrounding 2°.5x 2°.5 blocks. It just happens that both considerations yield the same elevation criteria. In fact, if we compare Method A with Method F, it might appear that for elevations above 8 km, the two methods yield results within 1 mgal. This result is fortuitous at best, and no conclusions may be drawn from such comparison.

Table 1

Comparison of Upward Continued Anomalies

Elev. (km)	(1) 36° 32' 30", -117°27' 30"					(2) 36° 33' 45", -117° 26' 15"						(3) 36° 34' 00", -117° 26' 00"					
	A	B	D	E	F	A	B	C	D	E	F	A	B	C	D	E	F
4	58.8	35.1	35.1	35.1	37.5	34.9	31.7	27.7	31.7	31.8	35.3	30.0	30.2	26.2	30.2	30.3	34.0
5	41.3	29.4	29.4	29.4	31.7	29.5	26.6	23.3	26.6	26.6	29.5	26.5	25.4	22.1	25.4	25.4	28.3
6	31.5	24.8	24.8	24.9	27.0	24.8	22.4	19.6	22.4	22.4	24.7	22.8	21.4	18.6	21.4	21.4	23.8
7	25.2	21.1	21.1	21.2	23.0	20.9	18.9	16.6	18.9	19.0	20.9	19.5	18.0	15.7	18.0	18.1	20.0
8	20.8	18.1	18.1	18.1	19.7	17.6	16.0	14.1	16.0	16.1	17.7	16.5	15.3	13.3	15.3	15.3	16.9
9	17.4	15.5	15.5	15.6	17.0	14.9	13.6	12.0	13.6	13.7	15.0	14.0	12.9	11.3	12.9	13.0	14.3
10	14.8	13.4	13.4	13.5	14.7	12.6	11.6	10.2	11.6	11.6	12.8	11.9	11.0	9.6	11.0	11.0	12.1
12	11.0	10.2	10.1	10.2	11.1	9.1	8.4	7.5	8.4	8.5	9.3	8.6	7.9	7.0	7.9	8.0	8.8
14	8.3	7.8	7.8	7.8	8.4	6.7	6.2	5.5	6.2	6.2	6.8	6.2	5.8	5.0	5.8	5.8	6.4
16	6.5	6.1	6.1	6.1	6.6	4.9	4.6	4.1	4.6	4.6	5.0	4.6	4.2	3.7	4.2	4.3	4.7

4.3 Accuracy of the Upward Continued Anomaly

It is of interest to know to what degree of reliability we can attach to the upward continued anomaly. In sections 4.1 and 4.2 two considerations have been made with respect to the subject: block size and integration requirements within a block. Of more pertinent interest is to consider the influence of the inaccuracy of the used gravity material and to question how far away from the computation point need the summation be carried out. Moritz [3,4] has extensively analyzed these problems so that we take from is work some pertinent equations.

4.31 Statistical Accuracy

In a previous report [7], the author outlined the following steps:

If the error covariance function of gravity anomalies is taken as,

$$\sigma(s) = \sigma_o \, e^{-c^2 s^2}$$

(7, equation numbers refer to the Moritz [4] report)

where σ_o, c^2 are constants and s is the anomaly separation. Moritz then shows that the standard error of the upward continued anomaly is,

$$m_H^2 = \frac{1}{8H^2} \; \frac{\sigma_o}{c^2} \tag{18}$$

Consider next the error of representation in anaanomaly block with sides a and b. Now is a point anomaly in this block is taken to represent the mean anomaly in the whole block, this error of representation is:

$$m^2 = \frac{\alpha^2}{6} \, (a^2 + b^2) \tag{42}$$

where α^2 is a constant found in the expression:

$$C(s) = C_o - \alpha^2 s^2 \tag{40}$$

$\alpha^2 = C_o/d^2$ where d is the separation at which C(r) equals $C_o/2$. C(r) is the covariance of the gravity anomalies and C_o is the variance of the anomalies. Moritz shows that:

$$m_H^2 = \frac{1}{H^2} \frac{A}{8\pi} m^2 \tag{48}$$

so that taking A = ab, the area of the square and m from (42) we have:

$$m_H^2 = \frac{1}{H^2} \frac{\alpha^2}{48\pi} ab\,(a^2 + b^2)$$

This equation should be evaluated for the given circumstances and area of uplifting. Some previous calculations show one estimate for this error using 5'x 5' squares could be:

$$m_H = \pm \frac{5}{H} \text{ mgals.}$$

Moritz obtained a similar but slightly smaller value.

4.32 Accuracy from Individual Block Accuracies

We have written the upward continuation formula in the form:

$$\Delta g_H = \frac{H}{2\pi} \sum\sum \Delta g \frac{dx\,dy}{D_o^3}$$

This equation can be regarded as simply a function composed of a summation of a certain number of elements. If we say that we know the accuracy of each of the anomalies ($m_{\Delta g}$) to be used in the summation, and neglecting the correlations between adjoining blocks due to non-independent estimation, we may write:

$$m_{\Delta g_H}^2 = \frac{H^2}{(4\pi)^2} \sum\sum \left(\frac{dx\,dy}{D_o^3}\right)^2 m_{\Delta g}^2$$

or in terms of the coefficient c

$$m_{\Delta g_H}^2 = \frac{1}{4\pi^2} \sum\sum c^2 m_{\Delta g}^2 .$$

This equation may be used in the computer program to be described simply by using the values of $m_{\Delta g}$ instead of Δg as input, and modifying several statements so that $c^2 m^2_{\Delta g}$ is computed instead of $c\Delta g$.

4.33 Maximum Accuracy

It has been previously mentioned that the original formulation of the upward continuation problem requires a summation over the whole earth sphere. When the problem is transferred to the plane the corresponding summation is over the entire plane. In practice it is not necessary to do such a complete summation since the values of c become smaller for distant areas. Moritz [3] has considered the question and his answer is that it is sufficient to go out to 10 times the elevation in order to be assured of an accuracy of ± 1 mgal. This figure assumes a mean anomaly of 10 mgals beyond the integration area. If this mean is smaller than specified, the accuracy of the upward continuation will decrease in direct proportions.

To be on the pessimistic side, though, we can accept the fact that if we go to a distance of 10 times the elevation the error committed would be no larger than ± 1 mgal. In some practical tests that have been made, we have added or deleted up to 30' blocks of anomalies beyond 10 times the elevation and found effects of only 0.1 or 0.2 mgals, thus leading to the belief that the figure of ± 1 mgal is somewhat pessimistic.

5. General Program Description and Flow Diagram

5.1 General Program Description

The computer program designed to perform the upward continuation of gravity anomalies is basically set up in two components. The first component basically reads in from cards the 5'x 5' mean anomalies in a block 30' wide in latitude extent and as long as needed (in longitude). This block is brought in and stored in a two dimensional array. After all cards in a block are stored, the anomalies are written on tape one row at a time (longitude extent). For each row, the latitude (deg. min.) longitude (deg., min., + east) and the number of anomalies are written. After each row the next row is written until all 5' strips in the 30' block are written. The program then returns for a completely new block and storage pattern. This continues until the supply of blocks is exhausted. This created tape (#9) is now rewound and is ready for use in calling the main body of the program which is in a subroutine: SUBS43.

The first part of SUBS43 reads all the data from the tape (#9) created in the first part of the program. All the anomalies are stored with information pertaining to the location of the first anomaly in the string and the number of anomalies. The advantage of such a method is that the location of each anomaly need not be specified. These 5'x 5' anomalies are stored in the array ANOM1.

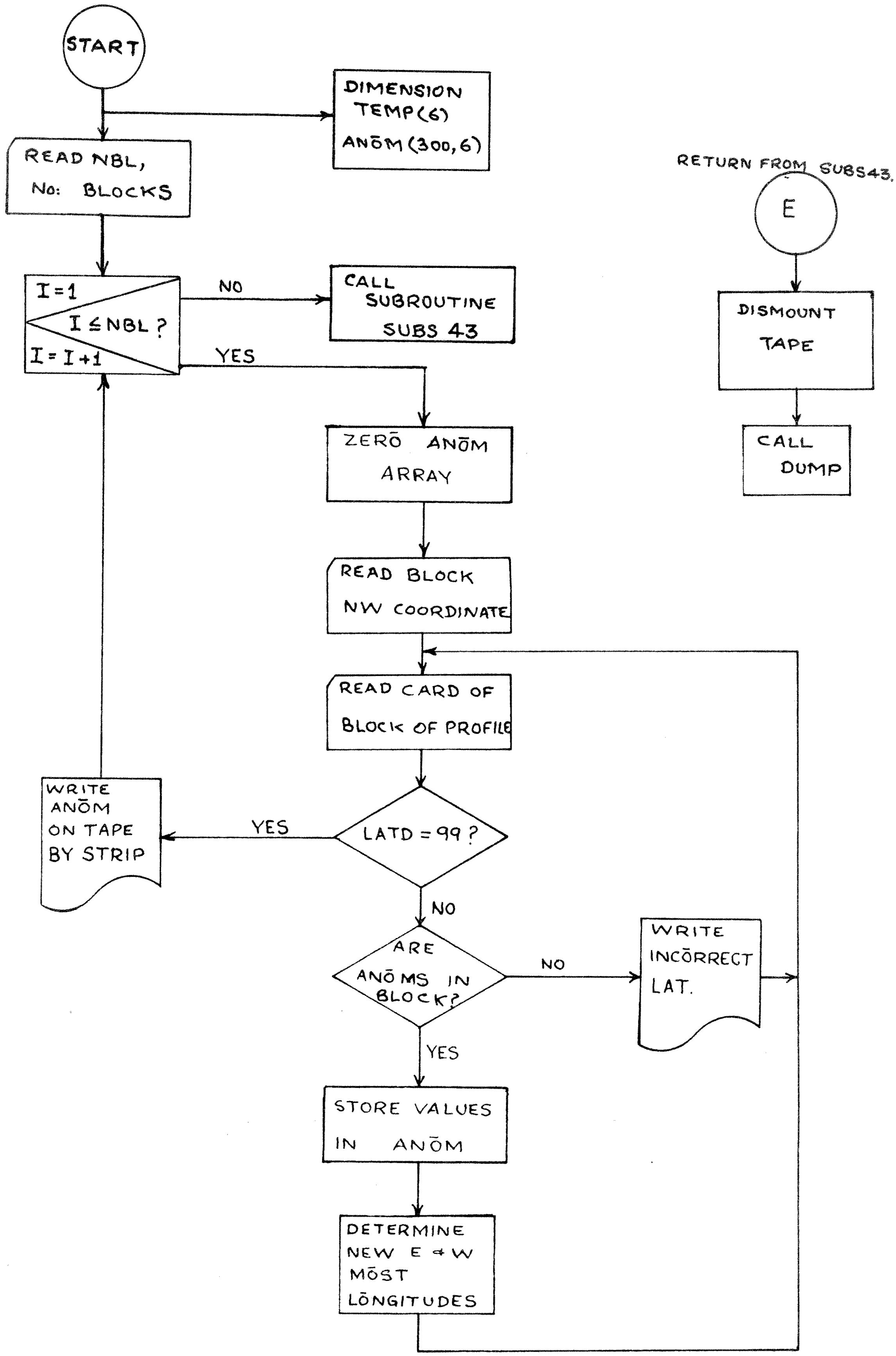
The machine is next informed if it is dealing with a latitude (999) or a longitude (9999) profile. This feature is described in a later section, but briefly it is a means to make computations and data handling more efficient by identifying a predominate north-south (999) or east-west (9999) profile. Now a card is read that contains four $2'\frac{1}{2}$x $2'\frac{1}{2}$ mean anomalies and the coordinates of the NW corner of the 5' square. This information is stored in the appropriate location in the array ANOM2. Consecutive cards having the same latitude (or longitude) are read in until the end of a strip is reached. Then a new strip is started and finished. This process is repeated until all the $2'\frac{1}{2}$ anomalies are in core. In addition to the proper storage of the anomalies, an additional computation is done to zero the 5'x 5' anomaly which is being duplicated by the $2'\frac{1}{2}$ information. In this manner, it is possible to combine both types of anomalies in the needed summation.

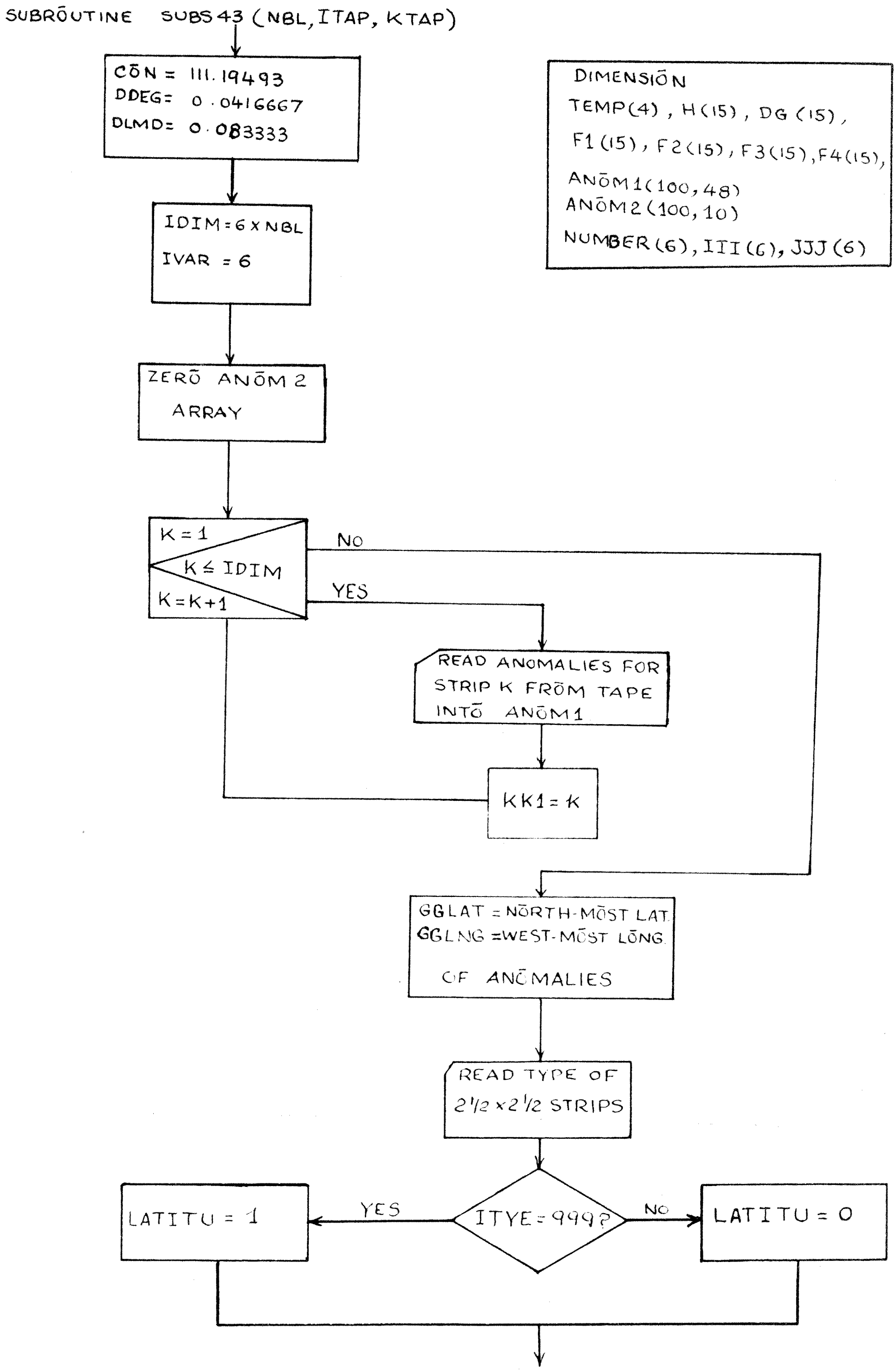
The computation then proceeds to read-in the location of a point at which the upward continued anomaly is to be computed. The location is specified by the latitude, longitude, number of elevations, and the sequence of elevations at which the anomaly is to be obtained. Thus it is possible to compute at several (up to 15 at present) elevations. This feature has the advantage of reducing the total amount of computation for the several elevations over the separate program submission for each elevation. In practice this means that little additional computer time is necessary for more than one elevation.

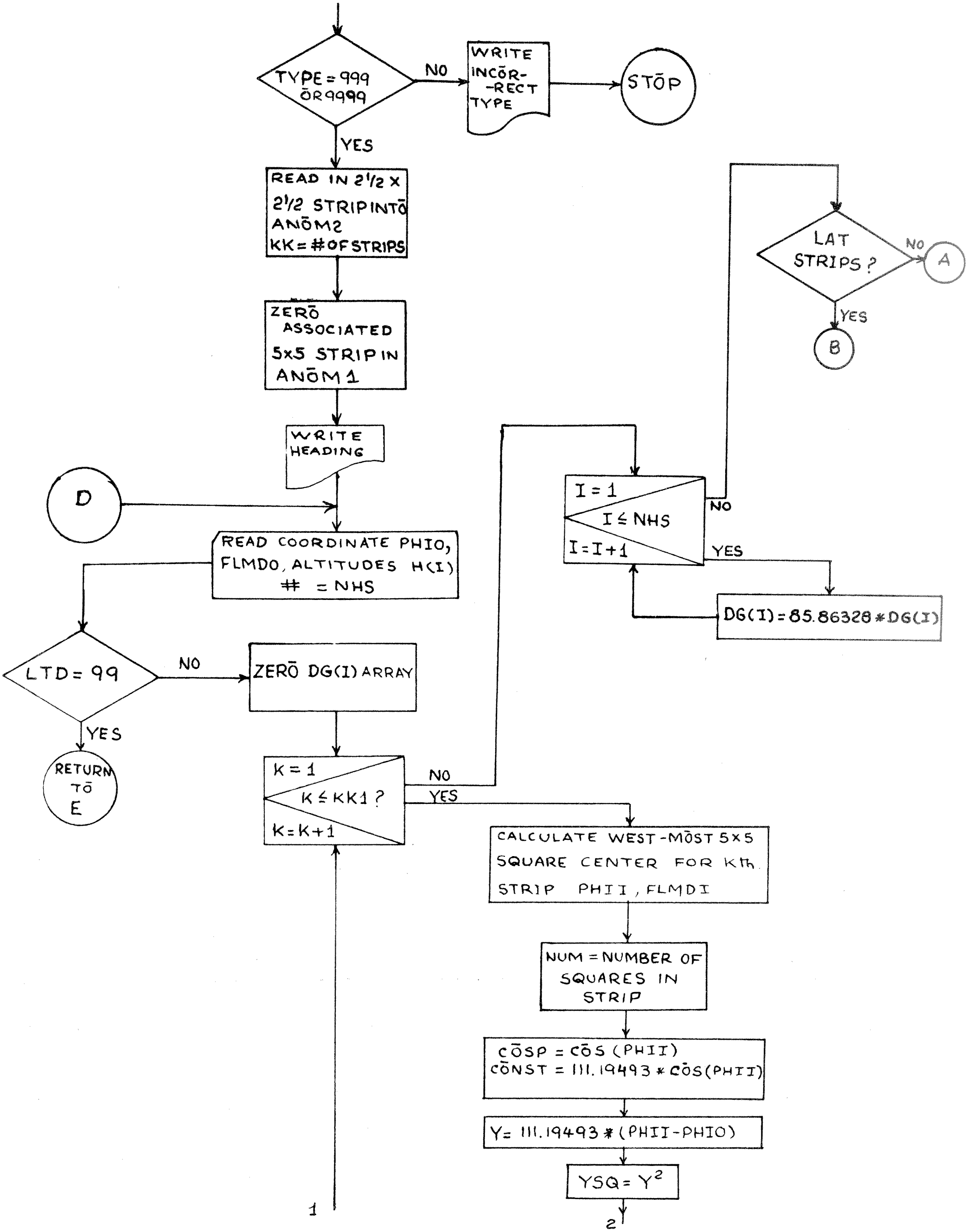
Having the coordinates, the first computation is the summation of the upward continued equation over the 5'x 5' mean anomalies. The next computation is the summation over the $2\frac{1}{2}$x $2\frac{1}{2}$ means using the integrated form of c_{ij}. Where the summation and computation takes place depends on whether we are dealing with an east-west or north-south profile. Before the print out of the final answers, the remaining step is to compute the normal gravity at the given latitude and elevation (regarded as above the reference ellipsoid). Finally the answers are given for the point: the latitude, longitude, elevations, with the anomaly and normal gravity for each elevation is printed out. All computations being completed for this point, the program returns for another point, repeating all computation until the point supply is exhausted.

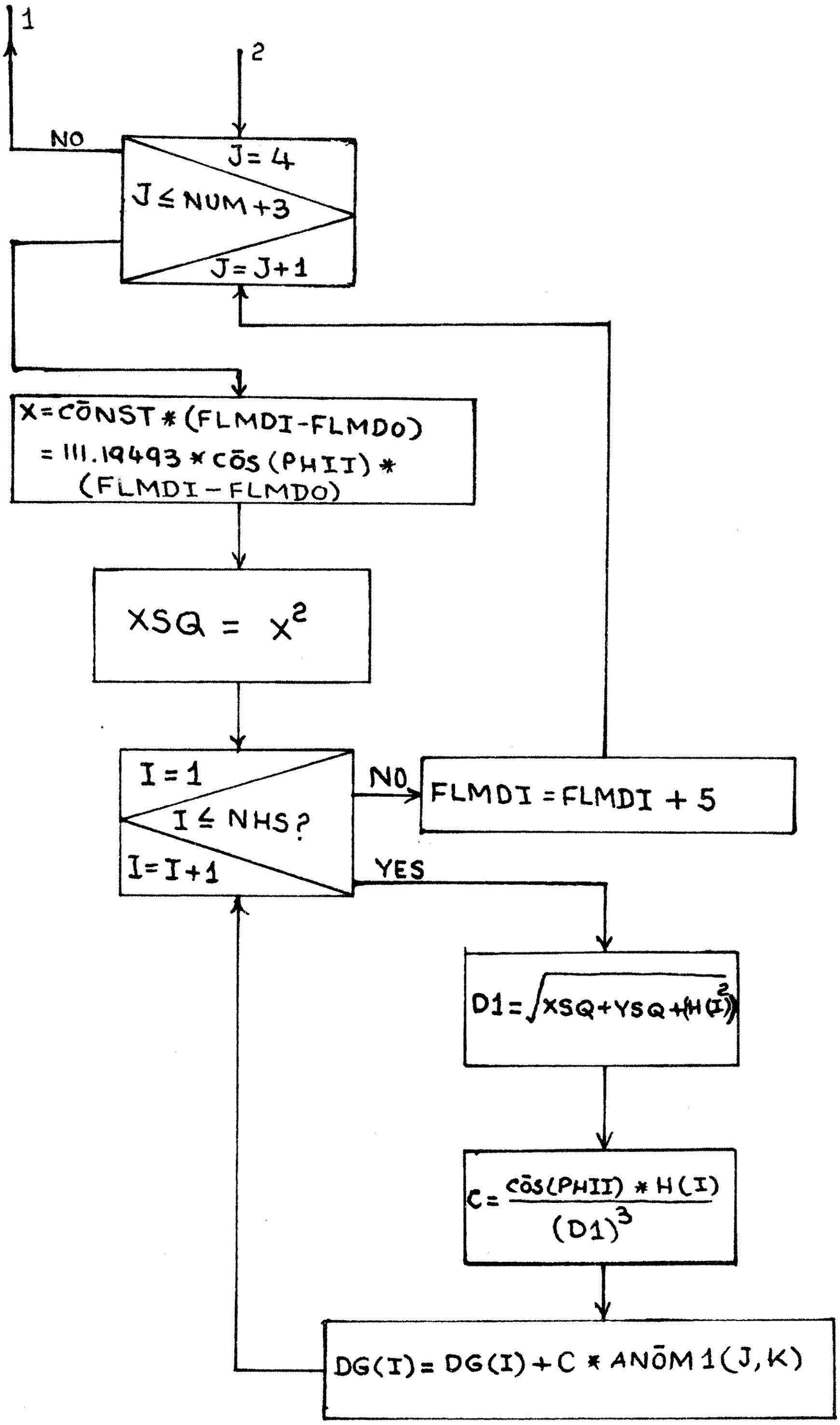
5.2 The Flow Diagram

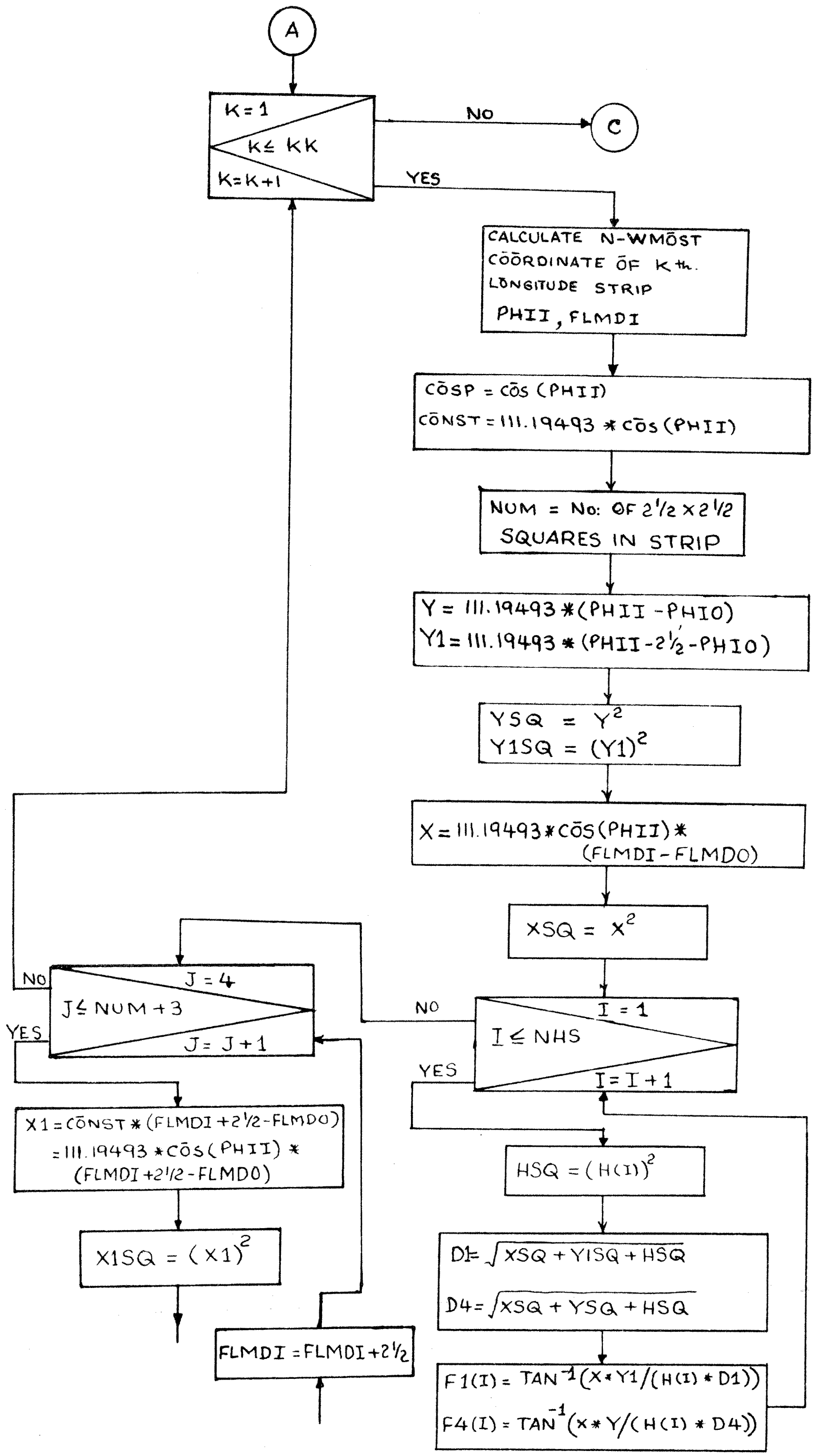
In order to show in a schematic way the operation of this program a flow diagram has been made and is given below. The symbols used in this diagram generally correspond to the ones that are found in the program. The flow diagram was designed by Doug Fleckner of the OSU Computation Center.

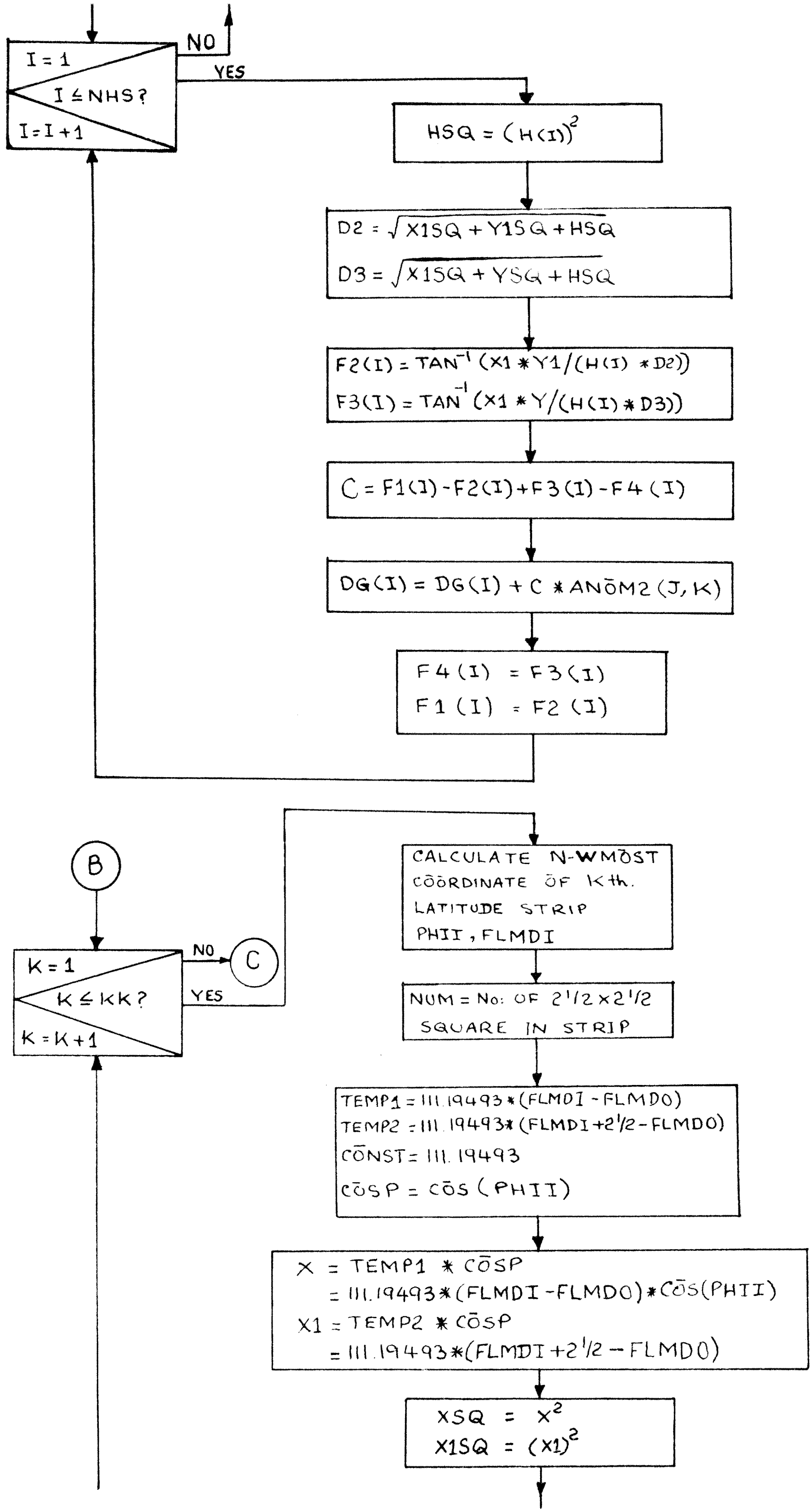

START
DIMENSION TEMP(6) ANŌM(300,6)
READ NBL, No: BLOCKS
I=1
I ≤NBL ?
I=I+1
NO
CALL SUBROUTINE SUBS 43
YES
ZERŌ ANŌM ARRAY
READ BLOCK NW COORDINATE
READ CARD OF BLOCK OF PROFILE
LATD = 99 ?
YES
WRITE ANŌM ON TAPE BY STRIP
NO
ARE ANŌMS IN BLOCK?
NO
WRITE INCŌRRECT LAT.
YES
STORE VALUES IN ANŌM
DETERMINE NEW E + W MŌST LŌNGITUDES
RETURN FROM SUBS43.
E
DISMOUNT TAPE
CALL DUMP

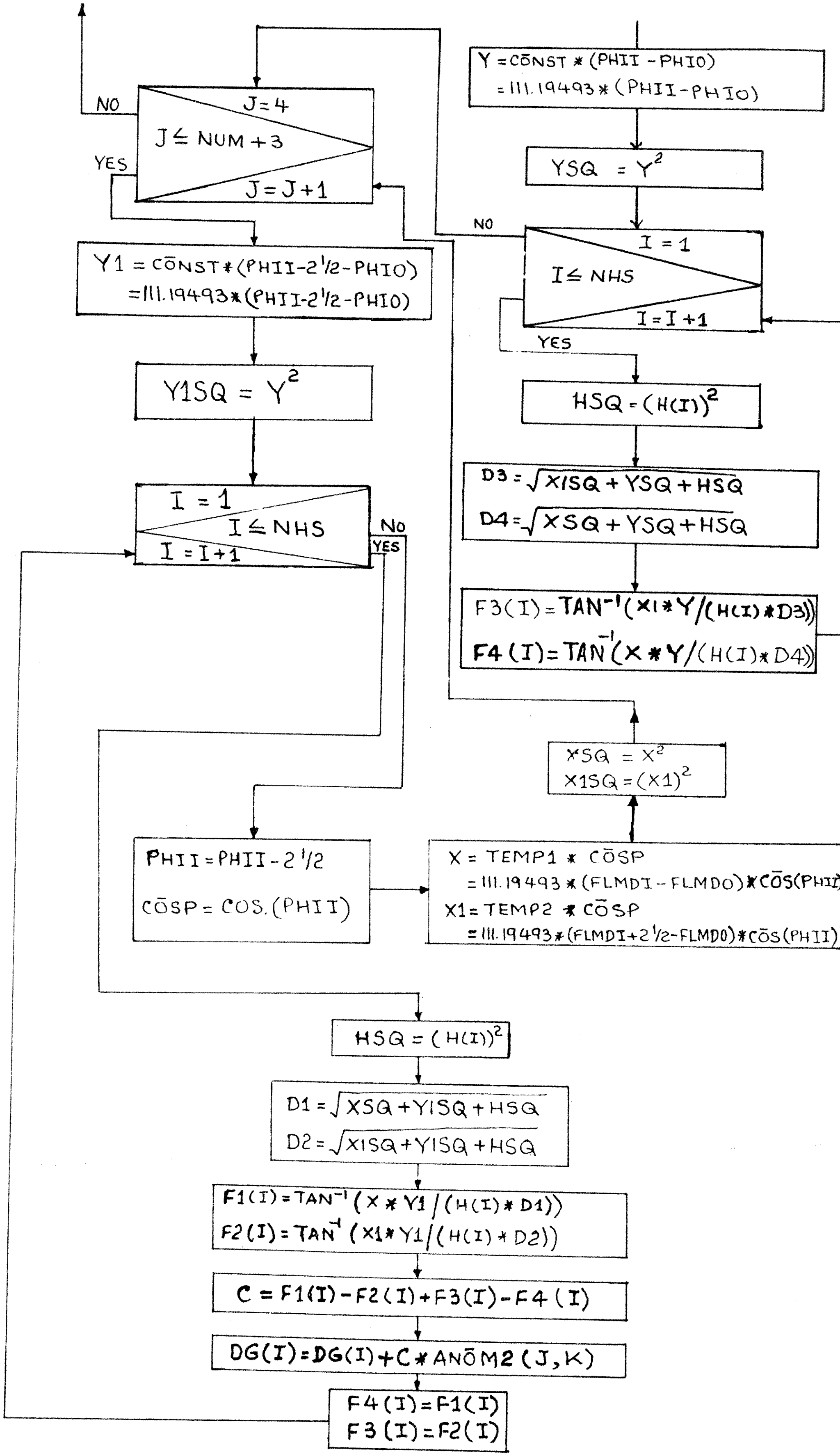

SUBROUTINE SUBS43 (NBL,ITAP,KTAP)
CON = 111.19493
DDEG= 0.0416667
DLMD= 0.083333
DIMENSION
TEMP(4), H(15), DG(15),
F1(15), F2(15), F3(15), F4(15),
ANOM1(100,48)
ANOM2(100,10)
NUMBER(6), III(6), JJJ(6)
IDIM=6xNBL
IVAR = 6
ZERO ANOM 2 ARRAY
K=1
K ≤ IDIM
K=K+1
NO
YES
READ ANOMALIES FOR STRIP K FROM TAPE INTO ANOM1
KK1 = K
GGLAT = NORTH-MOST LAT.
GGLNG = WEST-MOST LONG.
OF ANOMALIES
READ TYPE OF 2½x2½ STRIPS
ITYE=999?
YES
NO
LATITU = 1
LATITU = 0

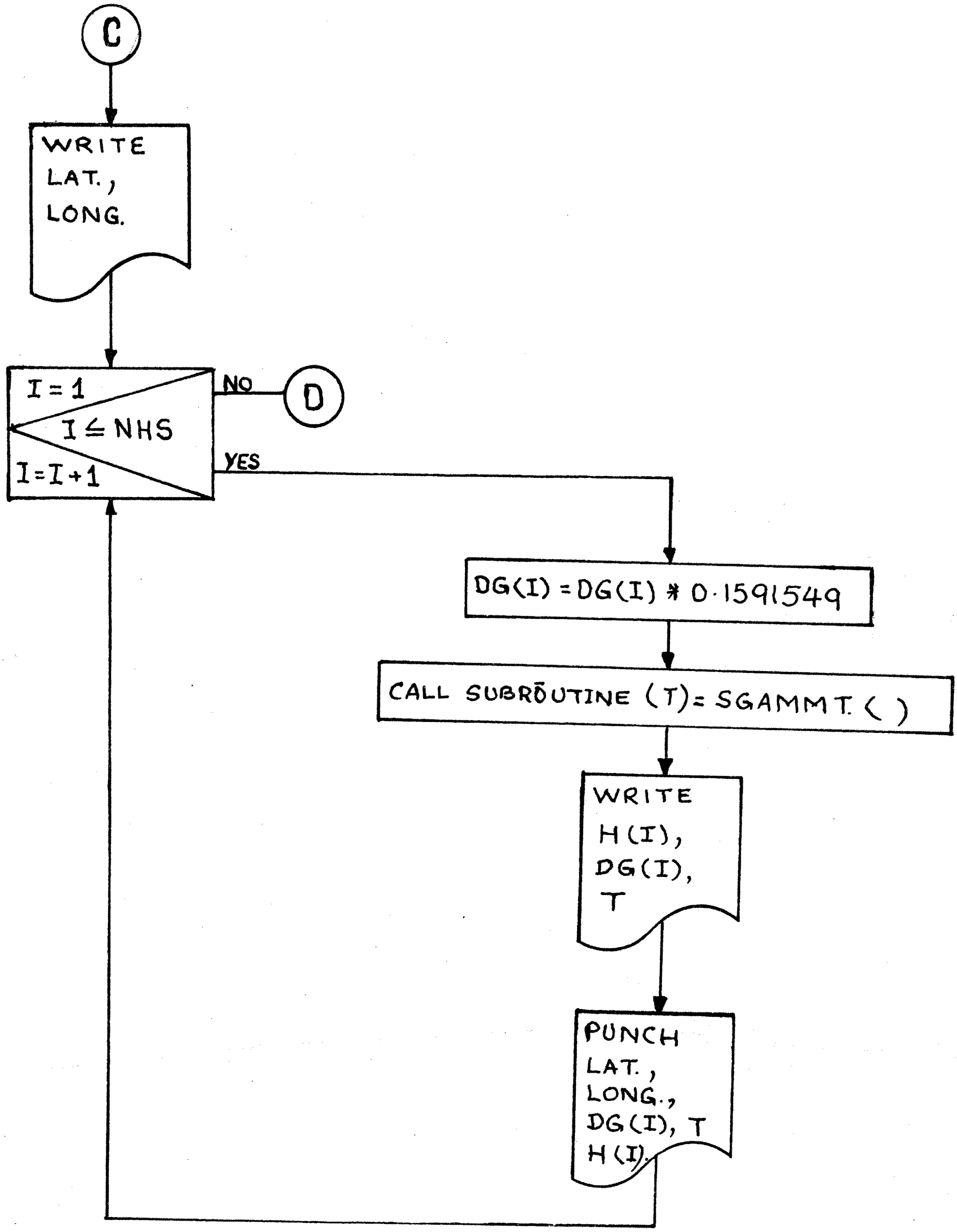

TYPE = 999 OR 9999
NO
WRITE INCORRECT TYPE
STOP
YES
READ IN 2 1/2 X 2 1/2 STRIP INTO ANOM2
KK = # OF STRIPS
ZERO ASSOCIATED 5x5 STRIP IN ANOM1
WRITE HEADING
D
READ COORDINATE PHIO, FLMDO, ALTITUDES H(I)
# = NHS
LTD = 99
NO
YES
RETURN TO E
ZERO DG(I) ARRAY
K = 1
K ≤ KK1 ?
K = K+1
NO
YES
I = 1
I ≤ NHS
I = I+1
NO
YES
DG(I) = 85.86328 * DG(I)
LAT STRIPS ?
NO
A
YES
B
CALCULATE WEST-MOST 5x5 SQUARE CENTER FOR Kth STRIP PHII, FLMDI
NUM = NUMBER OF SQUARES IN STRIP
COSP = COS (PHII)
CONST = 111.19493 * COS(PHII)
Y = 111.19493 * (PHII-PHIO)
YSQ = Y2
1
2


1
2
NO
J = 4
J ≤ NUM + 3
J = J + 1
X = CŌNST * (FLMDI - FLMDO)
= 111.19493 * CŌS (PHII) * (FLMDI - FLMDO)
XSQ = X²
I = 1
I ≤ NHS?
I = I + 1
NO
FLMDI = FLMDI + 5
YES
D1 = √(XSQ + YSQ + H(I)²)
C = CŌS(PHII) * H(I) / (D1)³
DG(I) = DG(I) + C * ANŌM1(J,K)


A
K=1
K≤ KK
K=K+1
NO
C
YES
CALCULATE N-WMOST COORDINATE OF Kth. LONGITUDE STRIP
PHII, FLMDI
COSP = COS (PHII)
CONST = 111.19493 * COS (PHII)
NUM = No: OF 2½ x 2½ SQUARES IN STRIP
Y = 111.19493 *(PHII - PHIO)
Y1 = 111.19493 * (PHII - 2½ - PHIO)
YSQ = Y²
Y1SQ = (Y1)²
X = 111.19493 * COS(PHII) * (FLMDI - FLMDO)
XSQ = X²
I = 1
I ≤ NHS
I = I + 1
NO
YES
HSQ = (H(I))²
D1 = √(XSQ + Y1SQ + HSQ)
D4 = √(XSQ + YSQ + HSQ)
F1(I) = TAN⁻¹(X*Y1/(H(I)*D1))
F4(I) = TAN⁻¹(X*Y/(H(I)*D4))
J = 4
J ≤ NUM+3
J = J+1
NO
YES
X1 = CONST * (FLMDI + 2½ - FLMDO)
= 111.19493 * COS(PHII) * (FLMDI + 2½ - FLMDO)
X1SQ = (X1)²
FLMDI = FLMDI + 2½


I = 1
I ≤ NHS?
I = I + 1
NO
YES
HSQ = (H(I))^2
D2 = √(X1SQ + Y1SQ + HSQ)
D3 = √(X1SQ + YSQ + HSQ)
F2(I) = TAN^-1 (X1 * Y1/(H(I) * D2))
F3(I) = TAN^-1 (X1 * Y/(H(I) * D3))
C = F1(I) - F2(I) + F3(I) - F4(I)
DG(I) = DG(I) + C * ANŌM2 (J, K)
F4 (I) = F3 (I)
F1 (I) = F2 (I)
B
K = 1
K ≤ KK?
K = K + 1
NO
YES
C
CALCULATE N-WMŌST CŌŌRDINATE ŌF Kth. LATITUDE STRIP PHII, FLMDI
NUM = No: OF 2 1/2 x 2 1/2 SQUARE IN STRIP
TEMP1 = 111.19493 * (FLMDI - FLMDO)
TEMP2 = 111.19493 * (FLMDI + 2 1/2 - FLMDO)
CŌNST = 111.19493
CŌSP = CŌS (PHII)
X = TEMP1 * CŌSP
= 111.19493 * (FLMDI - FLMDO) * CŌS(PHII)
X1 = TEMP2 * CŌSP
= 111.19493 * (FLMDI + 2 1/2 - FLMDO)
XSQ = X^2
X1SQ = (X1)^2

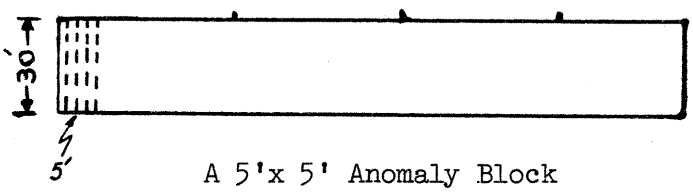

J=4
J ≤ NUM+3
J=J+1
NO
YES
Y = CONST * (PHII - PHIO)
= 111.19493 * (PHII-PHIO)
YSQ = Y^2
I = 1
I ≤ NHS
I = I+1
NO
YES
Y1 = CONST * (PHII-2 1/2-PHIO)
= 111.19493 * (PHII-2 1/2-PHIO)
Y1SQ = Y^2
HSQ = (H(I))^2
D3 = √(X1SQ + YSQ + HSQ)
D4 = √(XSQ + YSQ + HSQ)
I = 1
I ≤ NHS
I = I+1
NO
YES
F3(I) = TAN^-1(X1*Y/(H(I)*D3))
F4(I) = TAN^-1(X*Y/(H(I)*D4))
XSQ = X^2
X1SQ = (X1)^2
PHII = PHII - 2 1/2
COSP = COS.(PHII)
X = TEMP1 * COSP
= 111.19493 * (FLMDI - FLMDO) * COS(PHII)
X1 = TEMP2 * COSP
= 111.19493 * (FLMDI+2 1/2-FLMDO) * COS(PHII)
HSQ = (H(I))^2
D1 = √(XSQ + Y1SQ + HSQ)
D2 = √(X1SQ + Y1SQ + HSQ)
F1(I) = TAN^-1(X * Y1/(H(I) * D1))
F2(I) = TAN^-1(X1 * Y1/(H(I) * D2))
C = F1(I) - F2(I) + F3(I) - F4(I)
DG(I) = DG(I) + C * ANOM2(J,K)
F4(I) = F1(I)
F3(I) = F2(I)


C
WRITE LAT., LONG.
I = 1
I ≤ NHS
I=I+1
NO
D
YES
DG(I) = DG(I) * 0.1591549
CALL SUBROUTINE (T)= SGAMMT ( )
WRITE H(I), DG(I), T
PUNCH LAT., LONG., DG(I), T H(I)

6. The FORTRAN Program

6.1 Preliminary Comments

Before the specific input formats of the program are discussed, it is appropriate to describe a few preliminary points. The program is designed to accept two kinds of cards containing mean free-air anomalies. These cards will contain 5'x 5' and 2!5x 2!5 means. The 5' cards are handled in blocks; the latitude extent is 30' and the longitude extent is the complete extent of coverage desired for that latitude range. Each block consists of a sequence of cards that contain (on each card) 6 anomaly values along 30 minutes of latitude at a constant longitude. Each anomaly value then represents the anomaly in a 5'x 5' square whose longitude is specified on the card, and whose latitude may be computed from the latitude punched on the card. The coordinates of the square refer to the northwest corner of the square with the longitude being considered from 0^{o} to 360^{o} (positive east). If a block is 5^{o} wide, there will be 5 x 12 = 60 cards with a total of 60 x 6 = 360 anomalies. Figure III represents this picture:



A 5'x 5' Anomaly Block

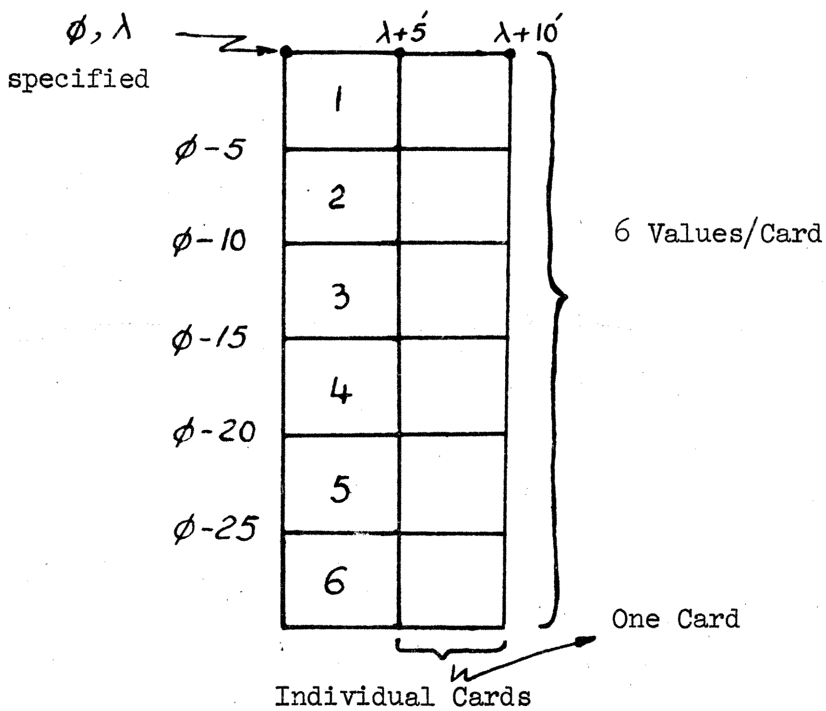


Figure III
5'x 5' Blocks

The format of each 5' card is (6F5.1 , 28X, 2(I3, I2)) with the 6 anomalies punched in the first 30 columns (5 cols/anomaly) and with the latitude and longitude (positive east) of the NW corner of the 30' strip appearing in columns 59-68. Specifically we have:

col.	value		
1-5	anomaly 1	see Figure III	
6-10	" 2		
11-15	" 3		
16-20	" 4		
21-25	" 5		
26-30	" 6		
59-61	latitude (deg)		of NW corner of strip
62-63	latitude (min)		
64-66	longitude (deg)	0°-360° (+ east)	
67-68	longitude (min)		

The number of 5'x 5' blocks to be used is particularly dependent on the elevations. As previously stated, the anomaly coverage should extend to a distance approximately 10 times the elevations from the computation point or profile. If a profile is used the 5' blocks should be chosen to border the profile as shown in Figure IV where an east-west profile is shown. Since the 5' blocks are 30' in latitude extent it is necessary at times to include more anomalies than actually needed but the only effect is a slightly increased computer running time.

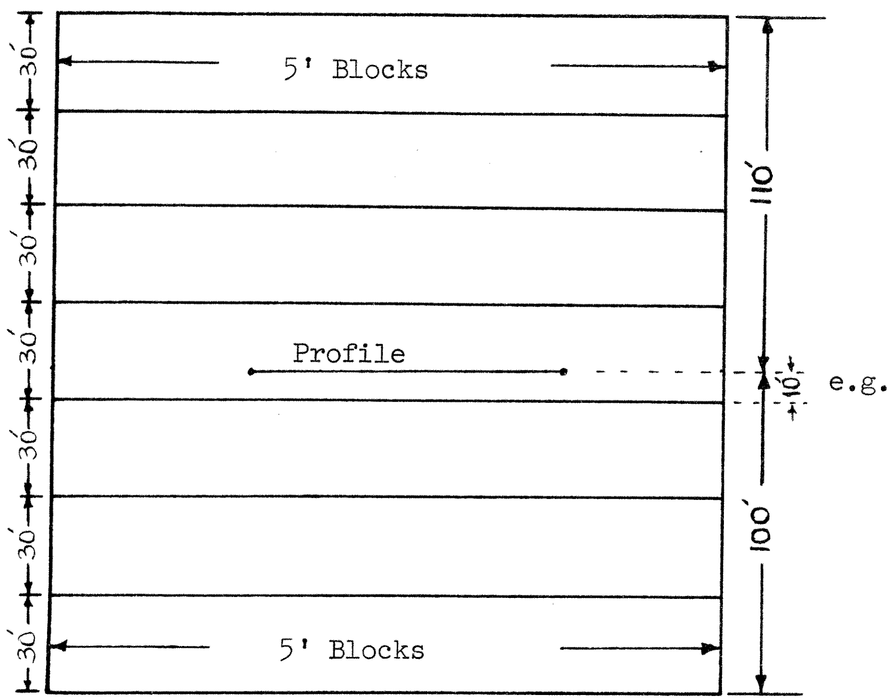
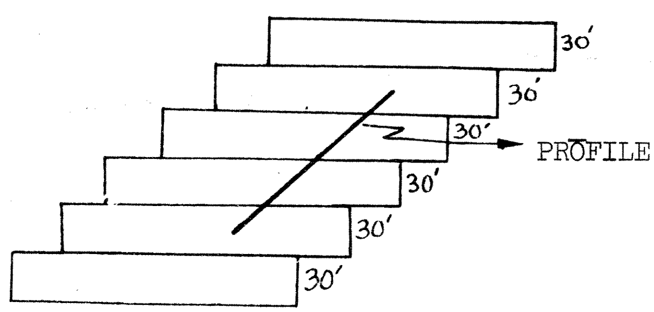


Figure IV

Example of Profile and 5'x 5' Block Relation

Similar examples could be shown for north-south and oblique profiles. With oblique profiles, the 5' blocks would not need to be chosen in a square fashion but may be set up in a stepped way as shown below.



Oblique Profile and 5'x 5' Block Relation

Figure V

In addition to the 5'x 5' mean anomalies that are used in the 5' blocks, the program also used the 2!5x 2!5mean anomalies that are especially important for low elevations (on the order of 7 or 8 km). These anomalies are needed only in the immediate vicinity of a computation point or profile. In order to define immediate vicinity we consider two cases. First consider an example when the computation point is situated at the center of a 5'x 5' square. In this case, 2!5x 2!5 anomalies are needed in the 5' square in which the point is situated and in the 8 adjacent 5'x 5' squares. This is shown in Figure VI.

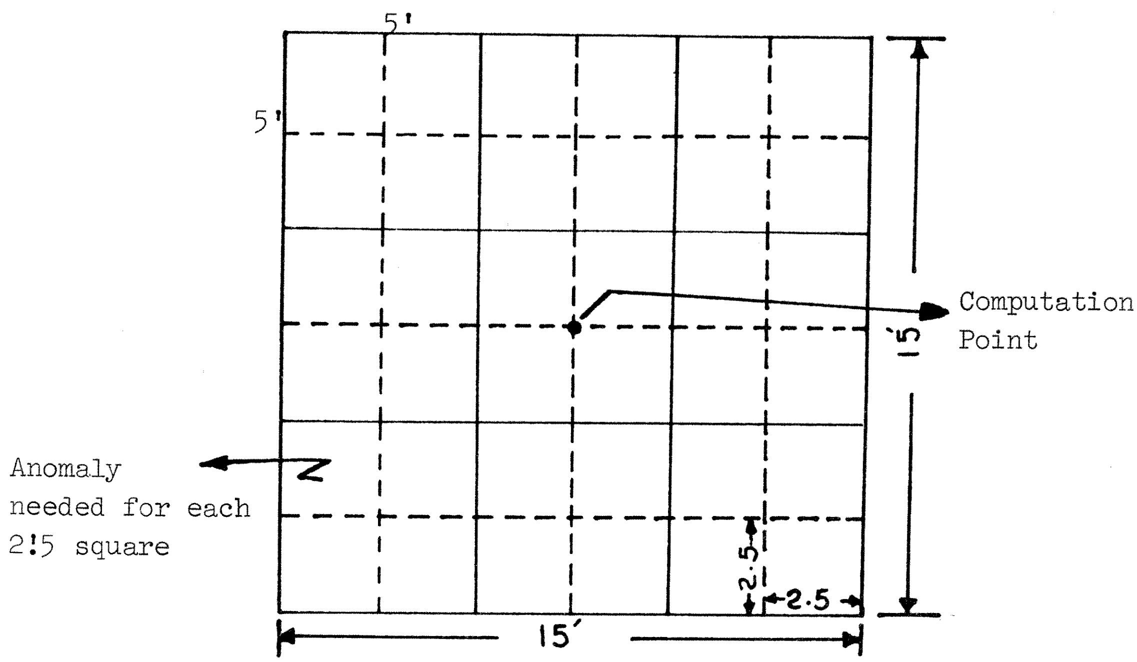


Figure VI

2!5 x 2!5 Anomalies

Surrounding Computation Point

A similar example may be set up for the needs of 2!5 mean anomalies along a profile. If we consider a profile along an integer 5' of latitude or longitude it is sufficient to use the 2!5 anomalies to 5' on each side of the anomaly and 5' at the ends. This is shown in Figure VII.

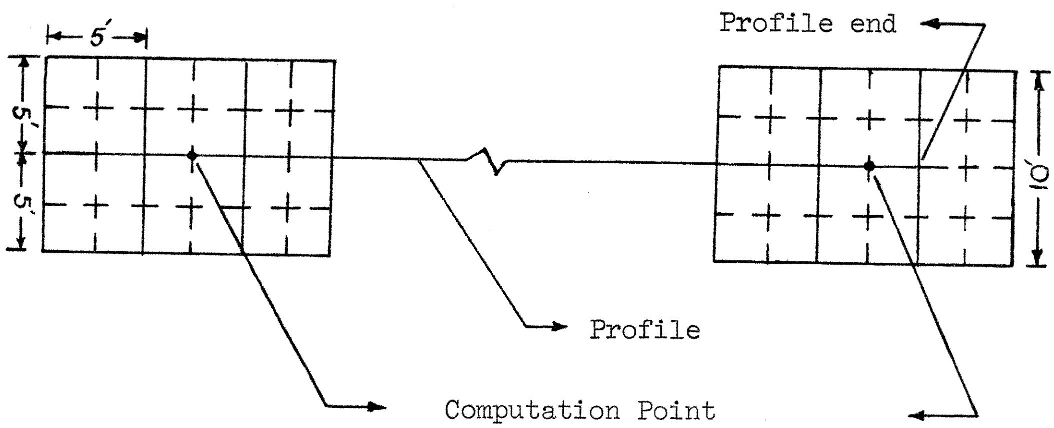


Figure VII

Location of 2!5 Mean Anomalies

on Profiles

Notice that along a profile, the computation point is taken at the center edge of a 5'x 5' square. This is usually done for symmetry purposes and is not necessary for elevations above 10 km, if at all.

The 2!5x 2!5 mean anomalies are punched four to a card representing the four 2!5 means in a 5'x 5' square. The first four fields on the card are the four mean anomalies in order 1, 2, 3, 4 with the latitude and longitude of the northwest corner of the 5' square being specified after the four mean anomalies. The four anomalies are punched in the first 24 columns, 6 cols/anomaly, with the latitude and longitude punched in columns 59-68. The card format is: (4F6.1, 34X, 2(I3, I2)). The order of the anomalies punched on the card is shown in Figure VIII.

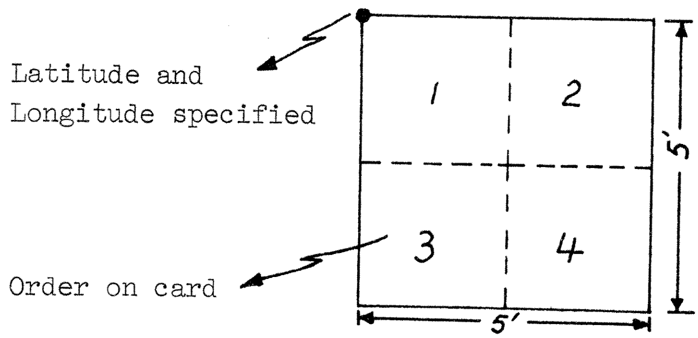


Figure VIII

Arrangement of 2'.5

Means for Punching

Within 5' Square

Specifically the format for the 2'.5x 2'.5 mean anomalies is:

col.	value		
1-6	anomaly 1		
7-12	" 2	see Figure VIII	
13-18	" 3		
19-24	" 4		
59-61	latitude (deg)	(- south)	
62,63	latitude (min)		of NW corner
64-66	longitude (deg)	0° to 360°(+ east)	of 5'x 5' block
67,68	longitude (min)		

For more efficient computer usage, it is necessary to specify a type of profile that is being computed. The profile type may be referred to as an east-west (or longitude) profile and a north - south (or latitude) profile. If an oblique profile is being used it is only necessary to describe it in its predominant sense. That is, it is predominantly east-west or predominantly north-south. If no predominant sense can be attached it is unimportant which type of profile it is called, but future input arrangements must be consistent with the profile types. Additionally, if only a small group of points (or just one point) is being computed, either east-west or north-south profile dèsignations may be used, with the input arrangements of the 2'.5 means in aggreement with the profile type to be described below.

Consider first the case of an east-west (longitude) profile which will be identified in the program by 9999. For this type, (9999) the 2'.5x 2'.5 cards should be read as follows: The first 2'.5 card represents the northwest corner of the strip. The second and succeeding cards at that latitude are read in order of increasing longitude (positive east) to the last card needed at that latitude. After this card comes the start of a new latitude, 5' minutes below the preceeding one. Though the starting longitudes are usually the same for the 2'.5 cards they need not be so. Again this set of cards is used to fill out a row of constant latitude along a varying longitude (increased by 5' for each card). If the profile is east-west, only two rows of cards with 2'.5 anomalies are needed. For oblique profiles more rows (i.e. different latitudes) with varying starting and ending longitude will be used. To summarize, the 2'.5 cards are used in order of highest latitude, and longitude first (NW corner of profile) followed by increasing the longitude and the starting latitude, following to the end of the planned profile, decrementing the latitude by 5' and starting again from the origin longitude. Further clarification is given in Figure IX.

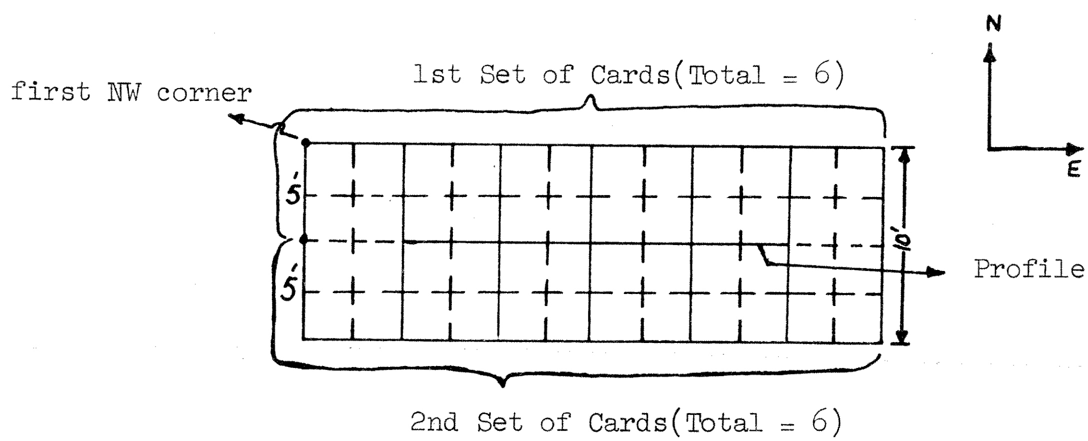


Figure IX
Example of Loading of 2'.5 Anomalies for East-West (9999) Profiles

A similar procedure is used for reading in the $2\overset{'}{.}5$ mean anomalies for north-south (indicated by 999) profiles. The first card is for the NW square used, followed by the cards for the same longitude, but with the latitude decreasing by 5' for each card. After a column is filled up (i.e., to cover areas needed for the profile), the column starting at the north edge is started at a longitude increased 5' from the preceeding. This is continued until the desired anomalies are read in. For the usual NS strip there will only be 2 vertical strips of cards. Further clarification is given in Figure X.

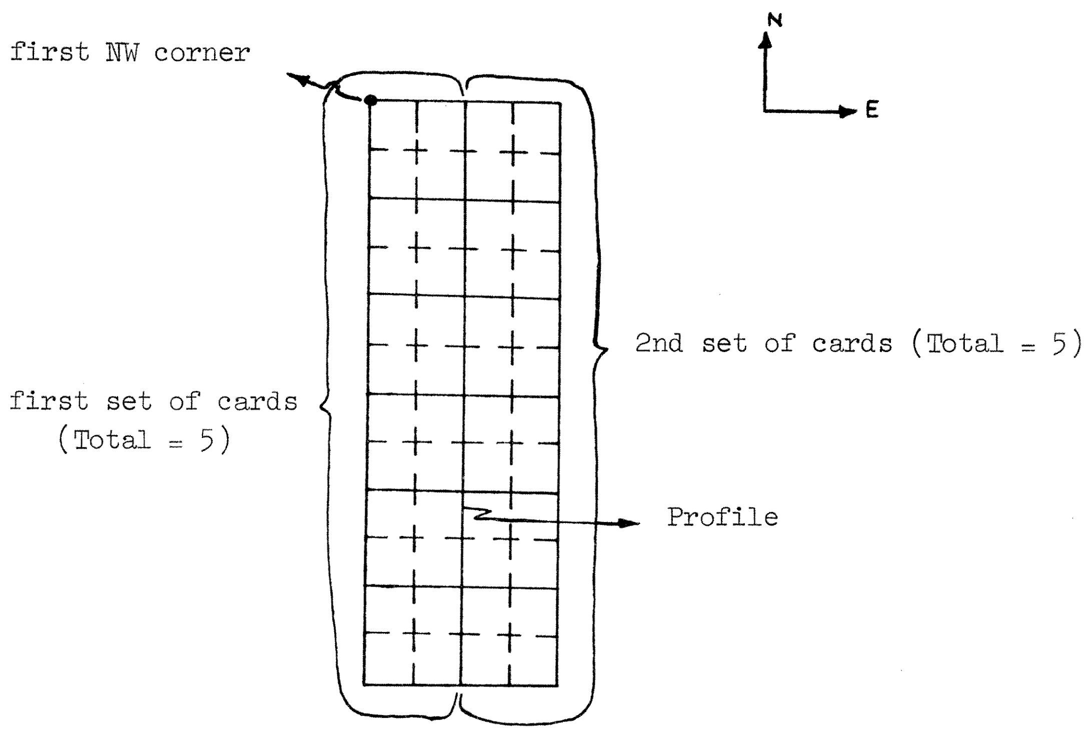


Figure X

Example of Loading of $2\overset{'}{.}5$ Anomalies for North-South (999) Profiles

6.2 Detailed Input Formats

In the previous sections we have outlined some of the steps and principles to be followed in the preparations of data for this program. Now it is necessary to become more specific. First we write down the information needed in a general form:

1. number of 30' 5'x 5' anomaly strips (NBL)

2. latitude, longitude of 30' strip
   5' cards in above strip
   (repeated for all necessary strips)

3. type of profile (999 or 9999)

4. needed 2!5x 2!5 mean anomalies

5. points and elevation at which the computation is to be made.

Specifically:

1. number of 30' 5'x 5' anomaly strips (NBL) format (I2)
   (col 1-2 = NBL)

For each 30' 5'x 5' block we have the following:

2. NW corner of 30' 5'x 5' block

   col:

1-2	NW corner of strip			(latitude - deg)
4-5	"	"	"	(latitude - min)
7-9	"	"	"	(longitude - deg, + east (0° to 360°))
11-12	"	"	"	(longitude - min)

The format for this card is : (I2, I3, I4, I3)
For the above specified strip, all 5'x 5' cards are entered.
The format for this card is: (6F5.1, 28X, 2(I3, I2))

col:

1-5	anomaly 1		
6-10	" 2		
11-15	" 3		
16-20	" 4		
21-25	" 5		
26-30	" 6		
59-61	latitude (deg) (- south)		of NW corner of strip
62-63	latitude (min)		
64-66	longitude (deg, + east (0° to 360°))		
67-68	longitude (min)		

This format has been described in detail in section 6.1. The last card in 30' 5'x 5' strip: 99 in col. 60-61. Step 2 is repeated for all NBL (specified in 1) blocks.

3. ITYPE = 999 for predominantly north-south profile; 9999 for predominantly east-west profile — see text for further discussion

col 1-4, format I4

4. 2!5x 2!5 mean anomalies
each card contains 4 anomalies in a 5'x 5' square.

col.

1-6	anomaly 1	see Figure VIII
7-12	" 2	
13-18	" 3	
19-24	" 4	
59-61	latitude (deg) (-south)	of NW corner of 5' block
62-63	latitude (min)	
64-66	longitude (deg) 0° to 360° (+ east)	
67-68	longitude (min)	

If ITYP = 999 (north-south strip) these cards are arranged in decreasing latitude (northern latitude first) within a given longitude, and in increasing (+ east) longitude (western most longitude first).

If ITYP = 9999 (east-west strip) these cards are arranged in increasing longitude (western most longitude first) within a given latitude, and in decreasing latitude (northern latitude first).

The last 2!5x 2!5 card is indicated by a 99 in col 60-61.

5. Computation point information:

latitude (deg, min, secs), longitude (deg, min secs),

number of elevations, elevations at which upward continuation is to take place.

for each point:

card a:

col:	value
1-2	lat (deg)
4-5	lat (min)
7-11	lat (secs)
12	long (sign) (+ or -)
13-15	long (deg) (+ east or - west)
17-18	long (min)
20-24	long (secs)
26-27	number of elevations
29-36	elevation 1
37-44	" 2
45-52	" 3
53-60	" 4
61-68	" 5

card b (if needed) remaining elevations (5 to a card)

card c (if needed) " " (" " ")

The format for card a is, (I2, I3, F6.2, A1, 2I3, F6.2, I3, 1X, 5F8.4).
The format for card b is 5F8.4.

The above information is repeated for all points to be used in the computation. The end of the run is signified by a card with a 99 in col 1-2.

6.21 Special Input Arrangements

There are two special cases that might arise in the use of the upward continuation program that could be of interest. They both pertain to the situation when we do not have any 2°.5x 2°.5 mean anomalies.

The first case is when elevations above 12 (or more safely 20km) are being considered. Thus we have seen that it is sufficient to consider the non-integrated form of the c_{ij}. Our input would then consist of the following:

1. sequence of 30' 5'x 5' anomaly strip

2. type of profile (either 999 or 9999, it is immaterial)

3. card with 99 in col. 60-61

4. computation point information

Note that no 2°.5x 2°.5 mean anomalies are used.

The second case arises when you are below 12 km (or more safely 20 km) where the c_{ij} in the integrated form must be considered. Without using the 2°.5x 2°.5 actual anomalies we can safely go down to 10 km. This is done by setting up a 2°.5x 2°.5 card with the same mean in each square as in the 5'x 5' square. These anomalies are used in the pertinent area. The input would look the same as the general case.

6.3 Output

In general the printed output consists of the following:

1. The original 5'x 5' mean anomalies in 5' strips in an east-west direction.

2. The profile designation

3. The 5'x 5' mean anomalies in 5' strips in an east-west direction in which the pertinent 5' squares have been zeroed due to having been replaced by 2'.5x 2'.5mean anomalies.

4. The 2'.5x 2'.5 mean anomalies in a form identifiable with the type profile designated .

5. The results under appropriate headings giving the latitude, longitude, and for all input elevation, the upward continued anomaly and the normal gravity corresponding to that latitude and elevation.

In more detail:

1. For every original 5'x 5' strip the following is printed:

   latitude, longitude, and number of anomalies in that strip.

The latitude and longitude refer to the northwest corner of the strip. This information is followed by the given anomalies starting from the given longitude and increasing in a positive (+ east) longitude direction.

2. Profile type

3. New 5'x 5' Anomaly Block - This is the same as the original set, but with the elements replaced by 2'.5x 2'.5 means being set zero. Above each set of anomalies is given the latitude (degs and decimals), longitude (degs and decimals), and number of anomalies.

4. The 2'.5x 2'.5 anomaly blocks are next given. A strip along a constant latitude or longitude are given. If ITYPE = 9999,

(east-west strip), the anomalies are given along a constant latitude. If ITYPE = 999 (north-south strips), the anomalies are given along a constant longitude. The coordinates of the NW corner of the profile are given above the anomalies themselves along with the number of 2.5 anomalies in that strip.

5. Finally the answers are given in terms of computation point coordinates, elevations, upward continued gravity and normal gravity at that latitude and elevation.

6.4 Space Allocation

In a program of this type it is important to identify the needed space that must be set aside in the program, and to establish how the size or needed space may be computed. The following paragraphs describe these subjects for those variables that are size dependent.

In the main program:

> ANOM(A, 6)
> A = (length of 30' block in degrees) times 12
> At the present time A = 300 so that (300/12 = 25°).

In the subroutine:

> SUBS43
> DG(B), F1(B), F2(B), F3(B), F4(B), H(B)
> B is the maximum number of elevations at which the upward continued anomaly is to be computed.
> At the present time, B = 15.
>
> ANŌM1(C,D)
> C = (length of 30' block in degrees) times 12 + 3
> D = (number of 30' blocks (NBL)) times 6
> At the present time, C = 100 so that (100-3)/12) = 8 degree band. D = 48 so that the number of 30' blocks is 8 or 4° in latitude extent.
>
> ANŌM2(E,F)
> E = maximum number of $2'\frac{1}{2}$ mean anomalies in any strip + 3
> F = number of $2'\frac{1}{2}$ strips to be used
> At the present time, E = 100, so ((100-3) = 97 $2'\frac{1}{2}$ anomalies) or 48 5'x 5' anomalies or essentially a 4° long band
> F = 12 $2'\frac{1}{2}$ strips or for a swath 30' wide.
>
> NUMBER(F), III(F), JJJ(F) should have the same space allocation given by the quantity F.

Additionally, if these variables are changed, so should several variables in the program: Thus: IVAR = F, IROW1 = C, IROW2 = E. In addition, some clearing operations identifiable at the beginning of the program and at the beginning of the subroutine must be carried through all allocated space.

7. Summary

The purpose of this report has been to describe and to put into operation the necessary steps needed for the computation of the upward continued gravity anomaly, Δg_H, at some elevation H. The operational requirement of the above purpose is satisfied through the enclosed digital computer programs written in FORTRAN II which enables the computation of Δg_H given the free-air anomalies on the surface of the geoid.

The equation that was used is essentially the Poisson Integral modified for use on a plane. The use of the plane is justified from the need for the anomalies only in the immediate vicinity of the computation point since the study was concerned only with heights that could be in the range found in airborne gravimetric work. The immediate vicinity is determined by going a distance, approximately 10 times the elevation at which the upward continuation is to take place, from the computation point.

Investigations were carried out to ascertain what the proper form of the equations to be used should take, and what size anomalies should be used for the upward continuation. It was found that accurate results could be obtained by using as integrated coefficient in the upward continuation equation for the squares within 5' of the squares in which the computation point fell, and in a non-integrated form for the other squares. It was also found that it is necessary to use not only 5'x 5' mean free-air anomalies, but $2'\frac{1}{2}$ x $2'\frac{1}{2}$ blocks must be used to maintain an accuracy of ± 1 mgal below 12 km in elevation. These anomalies are only those in the immediate (5') vicinity of the point. If the $2'\frac{1}{2}$ means are used, the lowest elevation at which the upward continued anomaly may be computed to maintain an accuracy of one mgal is on the order of 5 km. These figures were determined by computing the upward continued anomaly under various conditions with respect to equation coefficients and anomaly fields. Under different circumstances it is possible that slightly different elevation conditions could be warrented. However, it is felt that the values given should be reasonable guidelines to be used when actually carrying out the uplifting.

The computer program that was designed to be used for the upward continuation of the gravity anomalies has the possibility of using both $2'\frac{1}{2}$ and 5' anomaly blocks. Various control cards are used to identify each and insure their proper storage in the machine. In addition it is necessary to specify the computation as along a predominantly north-south or east-west direction. This feature improves computation efficiency when profiles are being uplifted but has no specific need for single or clustered computations. The time for the computation depends on the size of the ground area involved, the number of points to be uplifted, and the number of elevations per point. In a test case using a $3^{\circ} \times 5^{\circ}$ area of five minute anomalies (2160) and 36 $2'\frac{1}{2}$ means, the time required was 0.4 minutes for 10 elevations. This figure would be on the order of 0.3 minutes for 1 elevation. Thus the manner in which the program is written makes the time of execution only very slightly dependent on the number of elevations being used.

This study has presented an operational program to be used in the upward continuation of gravity anomalies to heights on the order of 50 km. In addition some suggestions are made on the limitations and applicability of such a program.

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[6] Rapp, R.H., A FORTRAN Program for the Computation of the Normal Gravity and Gravitational Field of the Earth, Report No.52, Dept. of Geodetic Science, OSU, Nov., 1965.

[7] Rapp, R.H., Gravity and Its Upward Continuation in the Area of Edwards Air Force Base, California, Report No. 55, Dept. of Geodetic Science, Ohio State University July, 1965.

APPENDICES

Appendix A

Source Statements for the Upward Continuation of Gravity Anomalies

The following program has been written in FORTRAN II operating under the monitor system at The Ohio State University Computer Center. In this installation the input tape is logical tape 5, which has been indicated by the variable ITAP in the program. The output tape is logical tape 6, which has been indicated by the variable KTAP in the program. In addition logical unit 9 is used internally in the program for intermediate storage.

The program is in single precision except for the subroutine which computes the normal gravity, this subroutine being in double precision. However, the output of this normal gravity is in single precision due to limitation of the OSU system. This is easily changed at installation where double precisions formats are available.

```
*       XEQ
*
*
*
C       UPWARD CONTINUATION OF GRAVITY ANOMALIES
        DIMENSIONTEMP(6),ANOM(300,6)
        REWIND9
        ITAP=5
        KTAP=6
        READINPUTTAPEITAP,13,NBL
  13    FORMAT(I2)
        DO51I=1,NBL
        DO19J=1,300
        DO19K=1,6
        ANOM(J,K)=0.0
  19    CONTINUE
        READINPUTTAPEITAP,11,LATAD,LATAM,LONGD,LONGM
  11    FORMAT(I2,I3,I4,I3)
        JFX=300
        JLX=1
  23    READINPUTTAPEITAP,12,(TEMP(K),K=1,6),LATD,LATM,LOD,LOM
  12    FORMAT(6F5.1,28X,2(I3,I2))
        IF(LATD-99)25,38,25
  25    IF(LATAD-LATD)29,26,29
  26    IF(LATAM-LATM)29,32,29
  29    WRITEOUTPUTTAPEKTAP ,30,(TEMP(K),K=1,6),LATD,LATM,LOD,LOM
  30    FORMAT(25H INCORRECT LATITUDE FOR  ,6F5.1,10X,2(I3,I2))
        GOTO23
  32    J=(LOD-LONGD)*12+(LOM-LONGM)/5+1
        DO34K=1,6
  34    ANOM(J,K)=TEMP(K)
        IF(JFX-J)36,36,35
  35    JFX=J
  36    IF(J-JLX)23,23,37
  37    JLX=J
        GO TO 23
  38    NUM=JLX-JFX+1
        LNM=LONGM+5*JFX-5
        ITEMP=LNM/60
        LND=LONGD+ITEMP
        LNM=LNM-ITEMP*60
        DO50K=1,6
        LTD=LATAD
        LTM=LATAM-5*K+5
        IF(LTM)47,49,49
  47    LTM=LTM+60
        LTD=LTD-1
  49    WRITETAPE9,LTD,LTM,LND,LNM,NUM
        WRITETAPE9,(ANOM(J,K),J=JFX,JLX)
  50    CONTINUE
  51    CONTINUE
        ENDFILE9
        REWIND9
        LAST=6*NBL
        WRITE OUTPUT TAPE KTAP,531
 531    FORMAT(1H1,10X,25HORIGINAL 5 BY 5 ANOMALIES)
        DO52L=1,LAST
        READTAPE9,LTD,LTM,LND,LNM,NUM
```

```
      READTAPE9,(ANOM(J,1),J=1,NUM)
   52 WRITEOUTPUTTAPEKTAP,53,LTD,LTM,LND,LNM,NUM,(ANOM(J,1),J=1,NUM)
   53 FORMAT(//10X,5I5/(10X,10F10.1))
      REWIND9
      CALLSUBS43(NBL,ITAP,KTAP)
      REWIND 9
      CALLEXIT
      END
      SUBROUTINESUBS43(NBL,ITAP,KTAP)
      DIMENSIONTEMP(4),DG(15),F1(15),F2(15),F3(15)
D     DIMENSIONH(15)
      DIMENSIONF4(15),ANOM1(100,48),ANOM2(100,12),NUMBER(12)
      DIMENSION III(12),JJJ(12)
      CON=111.19493
      DDEG=0.0416667
      DLMD=0.083333
D     FLMD0=0.0
D     PHI0=0.0
D     T=0.0
      IDIM=6*NBL
      IVAR=12
      IROW1=100
      IROW2=100
      REWIND9
      DO5I=1,100
      DO5J=1,10
    5 ANOM2(I,J)=0.0
      DO10K=1,IDIM
      READTAPE9,LTD,LTM,LND,LNM,NUM
      NUMP3=NUM+3
      READTAPE9,(ANOM1(J,K),J=4,NUMP3)
      T1=LTD
      T2=LTM
      ANOM1(1,K)=T1+T2/60.0
      T1=LND
      T2=LNM
      ANOM1(2,K)=T1+T2/60.0
      ANOM1(3,K)=NUM
   10 KK1=K
      GGLAT=ANOM1(1,1)
      GGLNG=ANOM1(2,1)
      READINPUTTAPEITAP,18,ITYPE
   18 FORMAT(I4)
      WRITEOUTPUTTAPEKTAP,17,ITYPE
   17 FORMAT(1H1,9X,19HTYPE OF PROFILE IS ,I6/10X,24HNEW 5 BY 5 ANOMALY
     1BLOCK)
      LATITU=0
      IF(ITYPE-999)21,19,20
   19 LATITU=1
      GO TO 24
   20 IF(ITYPE-9999)21,24,21
   21 WRITEOUTPUTTAPEKTAP,22
   22 FORMAT(10X,27HINCORRECT TYPE, JOB DELETED)
      GOTO209
   24 KK=1
      I=4
      J=1
      NEW=0
```

```
      IF(ANOM2(1,J))29,28,29
28    IF(ANOM2(1,J+1))29,30,29
30    NEW=1
29    READINPUTTAPEITAP,31,(TEMP(K),K=1,4),LTD,LTM,LND,LNM
31    FORMAT(4F6.1,34X,2(I3,I2))
      IF(LTD-99)32,1470,32
32    IF((J/2)*2-J)320,34,320
320   IF((I/2)*2-I)34,33,34
33    IF(I-3)34,34,330
330   IF(I-IROW1)331,331,34
331   IF(J)34,34,332
332   IF(J-IVAR)37,37,34
34    WRITEOUTPUTTAPEKTAP,35,(TEMP(K),K=1,4),LTD,LTM,LND,LNM
35    FORMAT(10X,43HINCORRECT 2 1/2 BY 2 1/2 CARD - JOB DELETED/10X,4F6.
     11,10X,4I4)
      GOTO209
37    IF(NEW)38,50,38
38    T1=LTD
      T2=LTM
      ANOM2(1,J)=T1+T2/60.0
      T1=LND
      T2=LNM
      ANOM2(2,J)=T1+T2/60.0
      IF(LATITU)41,41,44
41    ANOM2(1,J+1)=ANOM2(1,J)-DDEG
      ANOM2(2,J+1)=ANOM2(2,J)
      GOTO46
44    ANOM2(1,J+1)=ANOM2(1,J)
      ANOM2(2,J+1)=ANOM2(2,J)+DDEG
46    NUMBER(J)=0
      III(J)=(ANOM2(2,J)-GGLNG)/DLMD+3.5+1.0
      JJJ(J)=(GGLAT-ANOM2(1,J))/DLMD+0.5+1.0
      IF(III(J)-4)34,49,49
49    IF(JJJ(J))34,34,50
50    NUMBER(J)=NUMBER(J)+2
      ANOM2(I,J)=TEMP(1)
      ANOM2(I+1,J+1)=TEMP(4)
      IF(LATITU)54,54,59
54    ANOM2(I+1,J)=TEMP(2)
      ANOM2(I,J+1)=TEMP(3)
      NLONG=III(J)+(I-4)/2
      NLAT=JJJ(J)
      GOTO63
59    ANOM2(I+1,J)=TEMP(3)
      ANOM2(I,J+1)=TEMP(2)
      NLONG=III(J)
      NLAT=JJJ(J)+(I-4)/2
63    IF(NLONG-IROW2)64,64,34
64    IF(NLAT-IDIM)65,65,34
65    ANOM1(NLONG,NLAT)=0.0
      READINPUTTAPEITAP,31,(TEMP(K),K=1,4),LTD,LTM,LND,LNM
      IF(LTD-99)67,1470,67
67    IF(LATITU)680,680,1410
680   T1=LTD
      T2=LTM
      J=0.5+(ANOM2(1,1)-(T1+T2/60.0))/DDEG+1.0
      T1=LND
      T2=LNM
```

```
      I=3.5+(T1+T2/60.0-ANOM2(2,J))/DDEG+1.0
      GOTO1430
 1410 T1=LND
      T2=LNM
      J=0.5+(T1+T2/60.0-ANOM2(2,1))/DDEG+1.0
      T1=LTD
      T2=LTM
      I=3.5+(ANOM2(1,J)-(T1+T2/60.0))/DDEG+1.0
 1430 IF(J-KK)1440,1431,1431
 1431 KK=J+1
 1440 NEW=0
      IF(ANOM2(1,J))1443,1441,1443
 1441 IF(ANOM2(1,J+1))1443,1442,1443
 1442 NEW=1
 1443 IF(NEW)32,32,1450
 1450 I=4
      GOTO32
 1470 DO1490K=1,KK,2
      ANOM2(3,K)=NUMBER(K)
 1490 ANOM2(3,K+1)=NUMBER(K)
      DO 1495 K=1, IDIM
      NUMTMP = ANOM1(3,K)
      NUMTMP = NUMTMP+3
 1495 WRITEOUTPUTTAPEKTAP,1497,(ANOM1(J,K),J=1,NUMTMP)
 1497 FORMAT(1H0,19X,3F10.4/(10X,10F10.1))
      WRITEOUTPUTTAPEKTAP,1502
 1502 FORMAT(1H1,9X,28H2 1/2 BY 2 1/2 ANOMALY BLOCK   )
      DO1500K=1,KK
      NUMTMP=ANOM2(3,K)
      NUMTMP=NUMTMP+3
      WRITEOUTPUTTAPEKTAP,1501,(ANOM2(J,K),J=1,NUMTMP)
 1500 CONTINUE
 1501 FORMAT(1H0,19X,2F10.4,F5.0/(10X,10F10.3))
C
C     THE DATA IS ALL ENTERED, START CALCULATION
C
 68   WRITEOUTPUTTAPE6,69
 69   FORMAT(1H1,10X,8HLATITUDE,15X,9HLONGITUDE,12X,6HHEIGHT,9X,20HDELTA
     2 G H (IN MGALS),8X,14HNORMAL GRAVITY/8X,2(15HDEG   MIN    SEC ,8X))
 70   READINPUTTAPEITAP,71,LTD,LTM,FLTS,FLS,LND,LNM,FLNS,NHS,(H(I),I=1,N
     1HS)
 71   FORMAT(I2,I3,F6.2,A1,2I3,F6.2,I3,1X,(5F8.4))
      IF(LTD-99)73,209,73
 73   T1=LTD
      T2=LTM
      PHI0=T1+(T2+FLTS/60.0)/60.0
      T1=LND
      T2=LNM
      FLMD0=T1+(T2+FLNS/60.0)/60.0
8     FLSTMP=20000000000000*FLS
      IF(FLSTMP)75,74,75
 74   FLMD0=360.0-FLMD0
 75   CONTINUE
      DO76I=1,NHS
 76   DG(I)=0.0
      DO94K=1,KK1
      PHII=ANOM1(1,K)-DDEG
      FLMDI=ANOM1(2,K)+DDEG
```

```
      NUM    =ANOM1(3,K)
      COSP=COSF(PHII*0.0174532925)
      CONST=COSP*CON
      Y=CON*(PHII-PHIO)
      YSQ=Y*Y
      NUMP3=NUM+3
      DO93J=4,NUMP3
      X=CONST*(FLMDI-FLMDO)
      XSQ=X*X
      DO91I=1,NHS
      D1=SQRTF(XSQ+YSQ+H(I)**2)
      C=COSP*H(I)/(D1**3)
   91 DG(I)=DG(I)+C*ANOM1(J,K)
      FLMDI=FLMDI+DLMD
   93 CONTINUE
   94 CONTINUE
      DO96I=1,NHS
   96 DG(I)=85.86328*DG(I)
C
C     PROCEED TO CALCULATION FOR NEW ANOMS
C
      IF(ITYPE-999)104,142,104
C
C     THE CALCULATION IS FOR LONGITUDE CARDS
C
  104 DO137K=1,KK
      PHII=ANOM2(1,K)
      COSP=COSF(PHII*0.0174532925)
      CONST=COSP*CON
      NUM    =ANOM2(3,K)
      FLMDI=ANOM2(2,K)
      Y=CON*(PHII-PHIO)
      Y1=CON*(PHII-DDEG-PHIO)
      YSQ=Y*Y
      Y1SQ=Y1*Y1
      X=CONST*(FLMDI-FLMDO)
      XSQ=X*X
      DO121I=1,NHS
      HSQ=H(I)**2
      D1=SQRTF(XSQ+Y1SQ+HSQ)
      D4=SQRTF(XSQ+YSQ+HSQ)
      F1(I)=ATAN2F(X*Y1,H(I)*D1)
  121 F4(I)=ATAN2F(X*Y,H(I)*D4)
      NUMP3=NUM+3
      DO136J=4,NUMP3
      X1=CONST*(FLMDI+DDEG-FLMDO)
      X1SQ=X1*X1
      DO134I=1,NHS
      HSQ=H(I)**2
      D2=SQRTF(X1SQ+Y1SQ+HSQ)
      D3=SQRTF(X1SQ+YSQ+HSQ)
      F2(I)=ATAN2F(X1*Y1,H(I)*D2)
      F3(I)=ATAN2F(X1*Y,H(I)*D3)
      C=F1(I)-F2(I)+F3(I)-F4(I)
      DG(I)=DG(I)+C*ANOM2(J,K)
      F4(I)=F3(I)
  134 F1(I)=F2(I)
      FLMDI=FLMDI+DDEG
```

```
  136   CONTINUE
  137   CONTINUE
        GOTO186
C
C       THE CALCULATION IS FOR LATITUDE CARDS
C
  142   DO182K=1,KK
        NUM    =ANOM2(3,K)
        FLMDI=ANOM2(2,K)
        TEMP1=CON*(FLMDI-FLMD0)
        TEMP2=CON*(FLMDI+DDEG-FLMD0)
        PHII=ANOM2(1,K)
        CONST=CON
        COSP=COSF(PHII*0.0174532925)
        X=TEMP1*COSP
        X1=TEMP2*COSP
        XSQ=X*X
        X1SQ=X1*X1
        Y=CONST*(PHII-PHI0)
        YSQ=Y*Y
        DO161I=1,NHS
        HSQ=H(I)**2
        D3=SQRTF(X1SQ+YSQ+HSQ)
        D4=SQRTF(XSQ+YSQ+HSQ)
        F3(I)=ATAN2F(X1*Y,H(I)*D3)
  161   F4(I)=ATAN2F(X*Y,H(I)*D4)
        NUMP3=NUM+3
        DO181J=4,NUMP3
        Y1=CONST*(PHII-DDEG-PHI0)
        Y1SQ=Y1*Y1
        DO174I=1,NHS
        HSQ=H(I)**2
        D1=SQRTF(XSQ+Y1SQ+HSQ)
        D2=SQRTF(X1SQ+Y1SQ+HSQ)
        F1(I)=ATAN2F(X*Y1,H(I)*D1)
        F2(I)=ATAN2F(X1*Y1,H(I)*D2)
        C=F1(I)-F2(I)+F3(I)-F4(I)
        DG(I)=DG(I)+C*ANOM2(J,K)
        F4(I)=F1(I)
  174   F3(I)=F2(I)
        PHII=PHII-DDEG
        COSP=COSF(PHII*0.0174532925)
        X=TEMP1*COSP
        X1=TEMP2*COSP
        XSQ=X*X
        X1SQ=X1*X1
  181   CONTINUE
  182   CONTINUE
C
C       READY FOR OUTPUT
C
D186    RLMD0=0.0174532925*FLMD0
D       RPHI0=0.0174532925*PHI0
        IF(FLMD0-180.0)189,189,1881
 1881   FLMD0=FLMD0-360.0
  189   NEG=0
        IF(FLMD0)191,191,193
  191   NEG=1
```

```
      FLMDO=-FLMDO
  193 CONTINUE
      LDEG=FLMDO
      T1=LDEG
      LMIN=(FLMDO-T1)*60.0
      T2=LMIN
      SEC=(FLMDO-T1-T2/60.0)*3600.0
B     SIGN=200000000000
      IF(NEG)1991,198,1991
B1991 SIGN=400000000000
  198 IF(SEC-59.9951)291,289,289
  289 SEC=SEC-60.0
      LMIN=LMIN+1
  291 IF(LMIN-60)294,292,292
  292 LMIN=LMIN-60
      LDEG=LDEG+1
  294 IF(FLTS-59.9951)297,295,295
  295 FLTS=FLTS-60.0
      LTM=LTM+1
  297 IF(LTM-60)300,298,298
  298 LTM=LTM-60
      LTD=LTD+1
  300 CONTINUE
  199 WRITEOUTPUTTAPE6,200,LTD,LTM,FLTS,SIGN,LDEG,LMIN,SEC
  200 FORMAT(//8X,I3,I5,F7.2,7X,A1,I3,I5,F7.2)
      DO206I=1,NHS
      DG(I)=0.1591549*DG(I)
D     CALLSGAMMT(T,RPHI0,RLMDO,H(I)*1.0E5)
C     PUNCH205,LTD,LTM,FLTS,SIGN,LDEG,LMIN,SEC,H(I),T,DG(I)
C205  FORMAT(I2,I3,F6.2,L1,I4,I3,F6.2,F9.4,3PF10.2,0PF6.1)
  206 WRITEOUTPUTTAPE6,207,H(I),DG(I),T
  207 FORMAT(54X,F8.4,15X,F6.1,18X,3PF10.2)
      GOTO70
  209 REWIND9
      RETURN
      END
      SUBROUTINESGAMMT(SGAMT,FLAT,FLONG,ELEV)
D     OMEGSQ=0.5317494330504560E-8
D     C=0.5229760871395636E8
D     FKM=0.3986329044835964E21
D     Q0=0.7381303292625790E-4
D     A=637.838800E6
D     ESQ=0.00672267002233322
D     SINPHI=SINF(FLAT)
D     COSPHI=COSF(FLAT)
D     SINLAM=SINF(FLONG)
D     COSLAM=COSF(FLONG)
D     FN=A/(SQRTF(1.0-ESQ*SINPHI*SINPHI))
D     FNPH=FN+ELEV
D     X=FNPH*COSPHI*COSLAM
D     Y=FNPH*COSPHI*SINLAM
D     Z=FNPH*SINPHI-FN*ESQ*SINPHI
D     PSQ=X*X+Y*Y
D     P=SQRTF(PSQ)
D     CSQ=C*C
D     RSQ=PSQ+Z*Z
D     FKSQ=RSQ+CSQ
D     HSQ=SQRTF(FKSQ*FKSQ-4.0*PSQ*CSQ)
```

```
D         SSQALF=(FKSQ-HSQ)/(2.0*PSQ)
D         ALPHA=SQRTF(SSQALF/(1.0-SSQALF))
D         ALPHA=ATANF(ALPHA)
D         COSALF=COSF(ALPHA)
D         SINALF=SQRTF(SSQALF)
D         COTALF=COSALF/SINALF
D         BETA=SQRTF(PSQ)
D         BETA=ATANF(Z/(BETA*COSALF))
D         COSBET=COSF(BETA)
D         SINBET=SINF(BETA)
D         CSQBET=COSBET*COSBET
D         Q=0.5*(ALPHA-3.0*COTALF*(1.0-ALPHA*COTALF))
D         QP=(3.0*(1.0-ALPHA*COTALF)/SSQALF)-1.0
D         W=SQRTF(1.0-SSQALF*CSQBET)
D         GAMMA=(FKM*SSQALF/(CSQ*W))+(OMEGSQ*A*A*QP*SSQALF)/(2.0*C*Q0*W)*(SI
D        2NBET*SINBET-1.0/3.0)
D         GAMMB=(OMEGSQ*A*A*Q*SINALF*SINBET*COSBET)/(C*Q0*W)
D         GAMMT=SQRTF(GAMMA*GAMMA+GAMMB*GAMMB)
D         SGAMA=GAMMA-((OMEGSQ*C*CSQBET)/(W*SINALF/COSALF))
D         SGAMB=-GAMMB+((OMEGSQ*C*SINBET*COSBET)/(W*SINALF))
D         SGAMT=SQRTF(SGAMA*SGAMA+SGAMB*SGAMB)
D         RETURN
          END
```

Appendix B

Sample Input Data.

Included in the appendix are selected listings of the program input data. The first page contains a sample of the first input to the program where the number of 30' blocks has been specified and the first 30' block with 5' mean anomaly cards has started to be given. The second page shows the input connected with the $2'\frac{1}{2}$ mean anomalies and the specific coordinates at which the upward continued anomaly is to be computed.

6

38 0 240 0

224	-254	133	-72	-117	118	38 0240 0	1392
155	19	439	-106	28	400	38 0240 5	1393
132	456	526	21	248	149	38 024010	1394
226	833	792	11	170	349	38 024015	1395
1057	1449	793	88	685	446	38 024020	1396
1279	1620	594	90	850	708	38 024025	1397
911	950	761	795	1022	647	38 024030	1398
895	972	1136	863	1261	920	38 024035	1399
1454	1000	1221	1324	872	224	38 024040	1400
1784	1637	1405	1392	684	-83	38 024045	1401
308	90	450	877	550	193	38 024050	1402
-73	-32	314	611	521	504	38 024055	1403
-153	159	9	-141	407	810	38 0241 0	1464
288	434	174	-246	-82	1184	38 0241 5	1465
403	248	379	-371	-245	931	38 024110	1466
264	459	604	-4	69	454	38 024115	1467
203	540	554	458	333	66	38 024120	1468
118	-3	70	403	117	-257	38 024125	1469
288	-286	-238	-442	-615	-779	38 024130	1470
83	724	1161	-118	-927	-1129	38 024135	1471
573	860	1394	1292	406	49	38 024140	1472
116	37	-38	507	591	1317	38 024145	1473
255	-263	-524	-183	358	801	38 024150	1474
-215	-188	-565	-548	-529	331	38 024155	1475
-253	-270	-95	331	-25	-111	38 0242 0	1536
88	156	468	843	538	77	38 0242 5	1537
-192	81	670	670	487	521	38 024210	1538
-246	-141	329	-115	-59	453	38 024215	1539
-103	-155	-289	-323	-231	588	38 024220	1540
239	-52	-332	-255	-43	485	38 024225	1541
333	-42	-101	70	226	312	38 024230	1542
26	-179	2	548	295	5	38 024235	1543
-135	-222	-45	309	124	-22	38 024240	1544
-47	-175	-18	185	66	-81	38 024245	1545
-88	-122	124	192	-39	-141	38 024250	1546
-122	-182	13	-46	-146	276	38 024255	1547
-81	-96	47	-55	-167	45	38 0243 0	1608
30	-59	278	295	-41	28	38 0243 5	1609
-118	-145	13	190	166	54	38 024310	1610
-86	-128	-38	3	132	98	38 024315	1611
-141	-209	-134	-168	-192	-161	38 024320	1612
218	-55	-72	-103	-154	-178	38 024325	1613
510	15	-148	-284	-338	-413	38 024330	1614
-53	288	133	-3	-192	-371	38 024335	1615
-128	-86	-69	50	-56	-192	38 024340	1616
77	-140	-274	-299	-276	-388	38 024345	1617
11	335	-344	-268	-128	-85	38 024350	1618
-399	-322	-455	-309	-215	-34	38 024355	1619
-152	18	-182	-188	-278	-227	38 0244 0	1680
342	52	-234	-376	-398	-210	38 0244 5	1681
249	-126	-389	-440	-352	211	38 024410	1682
78	-195	-440	-431	-232	-62	38 024415	1683
172	206	-152	-118	247	-213	38 024420	1684

```
9999
  -16.0   20.0   40.0   50.0                                              +364024225
   39.6   40.0   60.0   30.0                                              +364024230
  -10.0    0.0   10.3    4.1                                              +364024235
   96.2   73.0   47.0   69.0                                              +363524225
   74.0   85.0   91.4   40.0                                              +363524230
   65.0   30.0   55.2  - 5.0                                              +363524235
   13.2   26.2  - 4.7  -25.1                                              +363024225
  -42.1   10.0  -40.1  -67.4                                              +363024230
   61.5   24.7   34.1   21.7                                              +363024235
                                                                            99
36 32 30.00-117 27 30.00 10 4.0000   5.0000   6.00000   7.0000   8.0000
   9.0000 10.0000 12.0000 14.0000 16.0000
36 33 45.00-117 26 15.00 10 4.0000   5.0000   6.00000   7.0000   8.0000
   9.0000 10.0000 12.0000 14.0000 16.0000
36 34 00.00-117 26 00.00 10 4.0000   5.0000   6.00000   7.0000   8.0000
   9.0000 10.0000 12.0000 14.0000 16.0000
99
```

Appendix C

Sample Output Data

Included in this appendix are selected listings from an actual execution of the program. The first page shows the form of the output of the original 5' mean anomalies. The second page shows the profile types and the modified 5' values (although in this case no zero elements appear). The third page gives the $2'\frac{1}{2}$ mean anomalies while the fourth and last page in this appendix given the anomalies and other identified information computed by the program.

ORIGINAL 5 BY 5 ANOMALIES

38 -0 240 -0 60

22.4	15.5	13.2	22.6	105.7	127.9	91.1	89.5	145.4	178.4
30.8	-7.3	-15.3	28.8	40.3	26.4	20.3	11.8	28.8	8.3
57.3	11.6	25.5	-21.5	-25.3	8.8	-19.2	-24.6	-10.3	28.9
33.3	2.6	-13.5	-4.7	-8.8	-12.2	-8.1	3.0	-11.8	-8.6
-14.1	21.8	51.0	-5.3	-12.8	7.7	1.1	-39.9	-15.2	34.2
24.9	7.8	17.2	6.1	13.3	-2.1	8.7	23.2	63.5	73.7

37 55 240 -0 60

-25.4	1.9	45.6	83.3	144.9	162.0	95.0	97.2	100.0	163.7
9.0	-3.2	15.9	43.4	24.8	45.9	54.0	-0.3	-28.6	72.4
86.0	3.7	-26.3	-18.8	-27.0	15.6	8.1	-14.1	-15.5	-5.2
-4.2	-17.9	-22.2	-17.5	-12.2	-18.2	-9.6	-5.9	-14.5	-12.8
-20.9	-5.5	1.5	28.8	-8.6	-14.0	33.5	-32.2	1.8	5.2
-12.6	-19.5	20.6	13.0	49.1	-5.5	8.3	26.6	46.4	32.8

37 50 240 -0 60

13.3	43.9	52.6	79.2	79.3	59.4	76.1	113.6	122.1	140.5
45.0	31.4	0.9	17.4	37.9	60.4	55.4	7.0	-23.8	116.1
139.4	-3.8	-52.4	-56.5	-9.5	46.8	67.0	32.9	-28.9	-33.2
-10.1	0.2	-4.5	-1.8	12.4	1.3	4.7	27.8	1.3	-3.8
-13.4	-7.2	-14.8	13.3	-6.9	-27.4	-34.4	-45.5	-18.2	-23.4
-38.9	-44.0	-15.2	18.9	23.8	-3.5	4.4	-1.6	6.5	4.8

37 45 240 -0 60

-7.2	-10.6	2.1	1.1	8.8	9.0	79.5	86.3	132.4	139.2
87.7	61.1	-14.1	-24.6	-37.1	-0.4	45.8	40.3	-44.2	-11.8
129.2	50.7	-18.3	-54.8	33.1	84.3	67.0	-11.5	-32.3	-25.5
7.0	54.8	30.9	18.5	19.2	-4.6	-5.5	29.5	19.0	0.3
-16.8	-10.3	-28.4	-0.3	5.0	-29.9	-26.8	-30.9	-18.8	-37.6
-44.0	-43.1	-11.8	22.3	35.7	28.9	18.0	-12.7	-3.7	9.9

37 40 240 -0 60

-11.7	2.8	24.8	17.0	68.5	85.0	102.2	126.1	87.2	68.4
55.0	52.1	40.7	-8.2	-24.5	6.9	33.3	11.7	-61.5	-92.7
40.6	59.1	35.8	-52.9	-2.5	53.8	48.7	-5.9	-23.1	-4.3
22.6	29.5	12.4	6.6	-3.9	-14.6	-16.7	-4.1	16.6	13.2
-19.2	-15.4	-33.8	-19.2	-5.6	-27.6	-12.8	-21.5	-27.8	-39.8
-35.2	-23.2	24.7	31.5	19.1	42.9	-14.3	-29.7	-18.8	-13.3

37 35 240 -0 60

11.8	40.0	14.9	34.9	44.6	70.8	64.7	92.0	22.4	-8.3
19.3	50.4	81.0	118.4	93.1	45.4	6.6	-25.7	-77.9	-112.9
4.9	131.7	80.1	33.1	-11.1	7.7	52.1	45.3	58.8	48.5
31.2	0.5	-2.2	-8.1	-14.1	27.6	4.5	2.8	5.4	9.8
-16.1	-17.8	-41.3	-37.1	-19.2	-38.8	-8.5	-3.4	-22.7	-21.0
21.1	-6.2	-21.3	-19.6	12.2	2.0	-29.9	-35.2	-19.1	18.8

37 30 240 -0 60

-14.8	-16.1	-26.2	30.1	24.5	52.7	50.3	19.6	36.1	23.5
62.8	156.6	109.1	143.2	120.4	116.9	-26.6	-73.5	-83.7	-95.0
-21.9	91.0	169.9	88.4	40.2	3.5	31.8	55.7	75.0	81.8
20.6	-4.6	-2.3	-10.0	-7.3	24.8	-3.3	2.2	0.7	-4.4
-16.4	-22.3	-22.0	-15.5	1.8	-13.2	24.7	33.2	-16.2	-16.2
23.3	30.1	-25.5	-39.8	-23.3	-1.1	-11.9	-49.4	-13.3	26.6

TYPE OF PROFILE IS 9999
NEW 5 BY 5 ANOMALY BLOCK

	38.0000	240.0000	60.0000						
22.4	15.5	13.2	22.6	105.7	127.9	91.1	89.5	145.4	178.4
30.8	-7.3	-15.3	26.8	40.3	26.4	20.3	11.8	28.8	8.3
57.3	11.6	25.5	-21.5	-25.3	8.8	-19.2	-24.6	-10.3	28.9
33.3	2.6	-13.5	-4.7	-8.8	-12.2	-8.1	3.0	-11.8	-8.6
-14.1	21.8	51.0	-5.3	-12.8	7.7	1.1	-39.9	-15.2	34.2
24.9	7.8	17.2	6.1	13.3	-2.1	8.7	23.2	63.5	73.7
	37.9167	240.0000	60.0000						
-25.4	1.9	45.6	83.3	144.9	162.0	95.0	97.2	100.0	163.7
9.0	-3.2	15.9	43.4	24.8	45.9	54.0	-0.3	-28.6	72.4
86.0	3.7	-26.3	-18.8	-27.0	15.6	8.1	-14.1	-15.5	-5.2
-4.2	-17.9	-22.2	-17.5	-12.2	-18.2	-9.6	-5.9	-14.5	-12.8
-20.9	-5.5	1.5	28.8	-8.6	-14.0	33.5	-32.2	1.8	5.2
-12.6	-19.5	20.6	13.0	49.1	-5.5	8.3	26.6	46.4	32.8
	37.8333	240.0000	60.0000						
13.3	43.9	52.6	79.2	79.3	59.4	76.1	113.6	122.1	140.5
45.0	31.4	0.9	17.4	37.9	60.4	55.4	7.0	-23.8	116.1
139.4	-3.8	-52.4	-56.5	-9.5	46.8	67.0	32.9	-28.9	-33.2
-10.1	0.2	-4.5	-1.8	12.4	1.3	4.7	27.8	1.3	-3.8
-13.4	-7.2	-14.8	13.3	-6.9	-27.4	-34.4	-45.5	-18.2	-23.4
-38.9	-44.0	-15.2	18.9	23.8	-3.5	4.4	-1.6	6.5	4.8
	37.7500	240.0000	60.0000						
-7.2	-10.6	2.1	1.1	8.8	9.0	79.5	86.3	132.4	139.2
87.7	61.1	-14.1	-24.6	-37.1	-0.4	45.8	40.3	-44.2	-11.8
129.2	50.7	-18.3	-54.8	33.1	84.3	67.0	-11.5	-32.3	-25.5
7.0	54.8	30.9	18.5	19.2	-4.6	-5.5	29.5	19.0	0.3
-16.8	-10.3	-28.4	-0.3	5.0	-29.9	-26.8	-30.9	-18.8	-37.6
-44.0	-43.1	-11.8	22.3	35.7	28.9	18.0	-12.7	-3.7	9.9
	37.6667	240.0000	60.0000						
-11.7	2.8	24.8	17.0	68.5	85.0	102.2	126.1	87.2	68.4
55.0	52.1	40.7	-8.2	-24.5	6.9	33.3	11.7	-61.5	-92.7
40.6	59.1	35.8	-52.9	-2.5	53.8	48.7	-5.9	-23.1	-4.3
22.6	29.5	12.4	6.6	-3.9	-14.6	-16.7	-4.1	16.6	13.2
-19.2	-15.4	-33.8	-19.2	-5.6	-27.6	-12.8	-21.5	-27.8	-39.8
-35.2	-23.2	24.7	31.5	19.1	42.9	-14.3	-29.7	-18.8	-13.3
	37.5833	240.0000	60.0000						
11.8	40.0	14.9	34.9	44.6	70.8	64.7	92.0	22.4	-8.3
19.3	50.4	81.0	118.4	93.1	45.4	6.6	-25.7	-77.9	-112.9
4.9	131.7	80.1	33.1	-11.1	7.7	52.1	45.3	58.8	48.5
31.2	0.5	-2.2	-8.1	-14.1	27.6	4.5	2.8	5.4	9.8
-16.1	-17.8	-41.3	-37.1	-19.2	-38.8	-8.5	-3.4	-22.7	-21.0
21.1	-6.2	-21.3	-19.6	12.2	2.0	-29.9	-35.2	-19.1	18.8
	37.5000	240.0000	60.0000						
-14.8	-16.1	-26.2	30.1	24.5	52.7	50.3	19.6	36.1	23.5
62.8	156.6	109.1	143.2	120.4	116.9	-26.6	-73.5	-83.7	-95.0
-21.9	91.0	169.9	88.4	40.2	3.5	31.8	55.7	75.0	81.8
20.6	-4.6	-2.3	-10.0	-7.3	24.8	-3.3	2.2	0.7	-4.4
-16.4	-22.3	-22.0	-15.5	1.8	-13.2	24.7	33.2	-16.2	-16.2
23.3	30.1	-25.5	-39.8	-23.3	-1.1	-11.9	-49.4	-13.3	26.6
	37.4167	240.0000	60.0000						
-27.5	-45.7	-48.4	-27.9	-28.4	18.6	53.8	-21.3	-52.6	64.1
86.3	62.8	47.7	117.6	144.2	111.0	71.4	-67.4	-91.3	-99.6
-51.5	85.5	105.4	23.6	45.3	-2.5	26.7	72.8	73.3	44.3
[illegible]	5.7	12.2	1.9	-11.6	-20.8	2.2	9.0	-4.4	[illegible]