A FORTRAN PROGRAM FOR THE UPWARD CONTINUATION OF GRAVITY ANOMALIES

by

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Prepared for

Air Force Cambridge Research Laboratories
Office of Aerospace Research
United States Air Force
Bedford, Massachusetts

Contract No. AF19(628)-5701
Project No. 7600
Task No. 760002,04
Scientific Report No. 5

The Ohio State University
Research Foundation
Columbus, Ohio 43212

February, 1966

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FOREWORD

This report was prepared by Dr. Richard H. Rapp, Assistant Professor, Department of Geodetic Science of The Ohio State University under Air Force Contract No. AF19(628)-5701, OSURF Project No. 2122. The contract covering the research is administered by the Air Force Cambridge Research Laboratories, Office of Aerospace Research, Lawrence G. Hanscom Field, Bedford, Massachusetts, with Mr. Owen Williams and Mr. Bela Szabo, Project Scientists.

The original upward continuation program was written by Doug Fleckner in the programming language SCATRAN used at the Ohio State University Computer Center. This program was then translated to the FORTRAN II version by Tom Tripp, also of the University Computer Center.

This report was submitted to the Air Force Cambridge Research Laboratories for review during February, 1966.
ABSTRACT

The equations of Moritz have been utilized in a computer program for the upward continuation of gravity anomalies. The solution for the anomaly at some elevation $H$ is obtained through the evaluation of the Poisson integral using the known anomalies in the surrounding areas on the ground. The input to the program consists of blocks of 5'x 5' mean free-air anomalies, as well as 21'5x 21'5 mean free-air anomalies for elevations below 12 km. The upward continued anomaly is thus computed for a sequence of points at, up to, 15 different input elevations. The lowest elevation that can be used is on the order of 4 km, while the highest elevation is limited by economical reasons to an elevation on the order of 50 km. In addition to the program itself numerous tests are described that pertain to square size and elevation limits to be used in the program.
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1. **Introduction**

The upward continuation of gravity anomalies may be defined as the computation of the gravity anomaly at some elevation $H$, given the gravity anomalies on the surface of the geoid, $H = 0$.

This problem has been examined by several authors in attempts to formulate theoretical solutions and to make practical computations. Two solutions are generally formulated: the first utilizes some form of a template; the second uses anomalies estimated in blocks of various sizes.

Template method investigations have been concerned especially with finding suitable compartment sizes or suitable points at which the gravity anomalies should be read on some grid surrounding the subpoint of the computation stations. Particular reference should be made to works of Henderson, Hirvonen, Peters, Tsuboi and others. The template method has the advantage that it maintains its usefulness for very low elevations, say on the order of 1 km or lower, because small compartments are utilized around the computation point. By using compartments at large distances from the computation point it is possible to extend the upward continued anomalies to the higher elevations, on the order of hundreds of kms. The disadvantage of the template method is that a point to point estimation of the mean anomalies in the various compartments is necessary. Thus considerable time and effort will be expended using the template method which should be critical for a production type evaluation of the upward continued anomaly. The template method is fundamentally manual and little utilizations may be made of a digital computer in obtaining the results.

The second method of upward continuation involves the use of mean anomalies in pre-determined sizes. These blocks are generally bordered by latitude and longitude grid lines. It is thus possible to estimate the mean values with no need for re-estimations when the computation point shifts. This is a distinct advantage over
the template method. However, there is some question on the size of the blocks. Mean anomalies may be usually found in $5^\circ \times 5'$, $1^\circ \times 1^\circ$ and $5'$x $5'$ blocks. These sizes are perhaps sufficient for the higher elevations but smaller size blocks are needed for lower elevations. This will require additional estimations near the computation point, but which in any event will be less, in most cases, than that required by the template method. Additional discussions concerning the block sizes needed will be found later in this report.

We thus choose the method of upward continuation which utilizes squares (or blocks) of various sizes. We shall limit the discussion and the computer program to those elevations (on the order of 200 kms) at which the earth may be approximated by a plane.

The theory and equation to be used in this report are not new. We will freely draw on the works of Hirvonen and Moritz in the discussion of this problem.
2. The Theory

2.1 Upward Continuation

The fundamental solution to the problem is found through the Poisson Integral:

\[ V_p = \frac{r^2 - R^2}{4\pi R} \int_s \frac{V_s}{D_s^3} \, d\sigma \]

\( V_p \): potential function at some point external to the sphere
\( r \): distance from center of sphere to the point
\( R \): radius of sphere
\( V_s \): function on the surface of the sphere (this function does not need to be a harmonic function)
\( D_s \): distance from the surface element \( V_s \) to \( P \)
\( d\sigma \): elemental area on the sphere
\( H \): elevation above the sphere \((r = R + H)\)

In terms of the gravity anomalies we have:

\[ V_p = r \Delta g_H \]

\[ V_s = R \Delta g \] where \( \Delta g_H \) is the anomaly at elevation \( H \) and \( \Delta g \) is the anomaly on the surface. Then (2.1) becomes:

\[ r \Delta g_H = \frac{r^2 - R^2}{4\pi R} \int_s \frac{R \Delta g}{D_s^3} \, d\sigma \]

or

\[ \Delta g_H = \frac{r^2 - R^2}{4\pi R} \int_s \frac{R \Delta g}{D_s^3} \, d\sigma \]

(2.2)
Equation (2.2) is usually transformed into another form by the substitution:

\[ t = \frac{R}{r} = \frac{R}{R + H} \]

Then:

\[ D_s^2 = R^2 + r^2 - 2Rr \cos \psi \]

\[ D_s^2 = r^2(1 + t^2 - 2t \cos \psi) \]

so

\[ D = rD_s \]

where \( D = (1 + t^2 - 2t \cos \psi)^{\frac{3}{2}} \) with \( \psi \), the spherical distance between the surface element and the computation point. In addition if we let \( d\sigma_1 \) be the area on a unit sphere:

\[ d\sigma = R^2 d\sigma_1 \]

we have:

\[ \Delta g_H = \frac{R^2(r^2 - R^2)}{4\pi r} \int d^3 \Delta g \ d\sigma_1 \]

or

\[ \Delta g_H = \frac{R^2}{r^2} \left(1 - \frac{R^2}{r^2}\right) \int d^3 \Delta g \ d\sigma_1 \]

or using \( t \):

\[ (2.3) \quad \Delta g_H = \frac{t^2(1 - t^2)}{4\pi} \int d^3 \Delta g \ d\sigma_1 \]

Equation (2.3) corresponds to equation (8.2) of Moritz [3] and is called by him the upward continuation integral. This equation may be used for the upward continuation of gravity anomalies.

There is, however, another form that may be obtained from (2.3) which is valuable in examining the case of plane assumptions. Using the substitution:
\[ t = \frac{R}{R + H} \]
\[ t^2 = \frac{R^2}{(R + H)^2} \]
\[ 1 - t^2 = \frac{(R + H)^2 - R^2}{(R + H)^2} = \frac{2H (R + H/2)}{(R + H)^2} \]
\[ D^3_s = \frac{D^3}{(R + H)^3} \]

we have:
\[ \Delta g_H = \frac{R^2}{4\pi} \frac{2H (R + H/2) (R + H)^3}{(R + H)^2} \int \int \int \frac{\Delta g}{D^3_s} \, d\sigma \]
\[ \Delta g_H = \frac{H}{2\pi} \frac{R + H/2}{R + H} \int \int \int \frac{\Delta g}{D^3_s} \, d\sigma \]

Now the ratio \( \frac{R + 2}{R + H} \) is close to 1: \( \sim 1 - \frac{H}{2R} \).

For the small elevations envisioned in these computations we will take this ratio to be one. At \( H = 200 \) km, the ratio is .985. We thus write the second form of the upward continued equation as:

\[ 2.4 \]
\[ \Delta g_H = \frac{H}{2\pi} \int \int \int \frac{\Delta g}{D^3_s} \, d\sigma \]

Equation (2.4) can also be transformed into a form suitable for use on a plane. These may be done by associating the distance \( D \) with a distance \( D \) on a plane and the spherical area element \( d\sigma_s \) with a plane area element \( dx \, dy \). If we thus set up a rectangular
coordinate system with origins at the subcomputation point, x axis positive east and y axis positive north we can write:

\[
\Delta g_H = \frac{H}{2\pi} \iint_D \Delta g \, dx \, dy
\]

where \[ D = x^2 + y^2 + H^2. \]

Equation (2.5) is the form of the upward continued integral in a plane approximation. Errors associated with it in comparison to the true equation limit its use. According to Moritz[3, p.71] this limit is for a distance up to 20° from the computation point for a height less than 250 km.

The computation of the x, y coordinate needs now to be discussed. The basic problem involves the transformation of some latitude, longitude coordinate difference on a sphere to a corresponding x, y coordinate on a plane. The obvious method would be some sort of map projection that would have minimum amounts of scale error and angular distortions within a range of the computation point. It is not necessary however to search for this because only simple approximations and equations may be used. For example Moritz suggests:

\[
\begin{align*}
\begin{cases}
  x = R \cos \varphi (\lambda - \lambda_p) \\
  y = R (\varphi - \varphi_p)
\end{cases}
\end{align*}
\]

where \( \varphi \) and \( \lambda \) are the coordinates of the particular surface point of interest and \( \varphi_p \) and \( \lambda_p \) are the coordinates of the actual computation point.

As the evaluation of (2.5) must be done as a finite summation we write:

\[
\Delta g_H = \frac{1}{2\pi} \sum \Delta g \, c(\varphi, \lambda)
\]

where the summation is to be taken over all the used anomalies and \( c(\varphi, \lambda) \) has two forms. The first is a direct form that is easily found from (2.5). The second is an integrated form. The latter is necessary for small elevations near the computation point.

The first form is:

\[
c(\varphi, \lambda) = \frac{H}{D^3} \\Delta A
\]
where $\Delta A$ is the area of the elemental surface block. $\varphi$ and $\lambda$ are taken as the coordinates of the center (mean coordinates) of the block under consideration. It can be seen that this form is satisfactory for more distant blocks but it will be sensitive to the latitude and longitude evaluations as the square is closer to the computation point. To this end Moritz computes an integrated form of $c(\varphi, \lambda)$. This is:

$$
\bar{c} (\varphi, \lambda) = \frac{1}{\Delta A} \iint c (\varphi, \lambda) \, dA
$$

$$
= \iint \frac{H \, dA}{D^3_0}
$$

where the integration is carried over the rectangle shown below;

For this square we have:

$$
\bar{c} (\varphi, \lambda) = \bar{c} (\varphi, \lambda) = \iint \frac{H \, dx \, dy}{(H^2 + x^2 + y^2)^{3/2}}
$$

Moritz gives the solution in the form:

$$
(2.9) \quad \bar{c} = F (1) - F (2) + F (3) - F (4)
$$

where:

$$
(2.10) \quad F (x, y) = \tan^{-1} \frac{xy}{HD^0}
$$
In (2.9) the value $F(i)$ indicates that these evaluations are to be done using (2.10) with the $x$, $y$ coordinates of point $i$ as indicated in the figure. The latitude used in the evaluation of the $x$ coordinate (i.e. in $\cos \varphi$) must be the same for point $(1,2)$ and $(3,4)$. This will then assure that the figure described by the coordinates is a rectangle as required by the derivation for $\bar{c}$. In fact this latitude is not critical and several variations are possible; for example, the northern latitude, the southern latitude or the mean latitude of the square in question. In the program which will be developed in this report, the northern latitude of the square is used.

The above completes the general discussion of the theory involved with the upward continuation of gravity anomalies.

2.2 Normal Gravity

Besides the interest in upward continuation of gravity anomalies, there is also some need for the normal gravity at the elevation, latitude, and longitude at which the upward continuation is to take place. The normal gravity can simply be defined as the sum of the attraction and centrifugal force of a point rigidly attached to a rotating ellipsoid of revolution whose surface is an equipotential surface.

Several authors have discussed this problem. A previous report [6] in this series has explained and summarized a solution due to Hirvonen. In that report the normal gravitation and normal gravity along several coordinate systems were computed. The specific need in this report is for only normal gravity in a direction perpendicular to the given equipotential surface of the reference ellipsoid. This direction is normally associated with the direction of the geodetic latitude, $\varphi$.

Since extensive discussion has taken place about the various methods, none will attempted here. However, since the normal gravity in the $\varphi$ direction is given in the computer program to follow, the specific equations to be used for this vertical component of normal gravity will be given in a later section.
3. Equations

3.1 Upward Continuation

The fundamental equation is:

$$\Delta g_H = \frac{H}{2\pi} \int \Delta g \, dx \, dy$$

or in a summation form,

$$\Delta g_H = \frac{1}{2\pi} \sum \Delta g \, c(x,y)$$

where the summation is carried out for each block containing a mean anomaly. In this program we will use 5'x 5' blocks and 2.5 x 2.5 blocks, so that the summation must be carried over each type of square. In addition the direct or non-integral form of $c(x,y)$ or $c(\phi,\lambda)$ as given in (2.8) will be used for the 5' squares. In this case the area of the 5' square would be computed as:

$$\Delta A = (R \Delta \phi R \cos \phi \Delta \lambda)$$

$$= \cos \phi \left( \frac{R \cdot 5'}{3437.468} \right)^2$$

$$\Delta A = 85.86328 \cos \phi \text{ where } R = 6371 \text{ km.}$$

For the smaller 2.5x 2.5 blocks that will be used close to the computation point the integrated form as given in (2.9) and (2.10) will be used. Further discussions will be found in a subsequent section of this report concerning these square sizes.

Given the following information:

$\phi_o', \lambda_o', H_1, H_2 \ldots H_k$ of the computation point
one set of 5'x 5' mean free-air anomalies $\Delta g(i,j)$
(center point coordinates $\phi_i', \lambda_i'$)
one set of 2.5x 2.5 mean free-air anomalies $\Delta g(i,j)$

Compute:

$$\Delta g_{H_1}, \Delta g_{H_2}, \ldots \Delta g_{H_k}$$
A. Sum over all 5'x 5' blocks (i,j):

For each block (i,j):

(3.1) \[ x = 111.19493 \cos \varphi_1 (\lambda_1 - \lambda_0) \]

(3.2) \[ y = 111.19493 (\varphi_1 - \varphi_0) \]

For each elevation (k), for each block:

(3.3) \[ D_{i,j,k}^2 = x^2 + y^2 + H_k^2 \]

(3.4) \[ \frac{c_{i,j,k}}{\Delta A} = \frac{\cos \varphi_1 H_k}{D_{i,j,k}^3} \]

(3.5) \[ \Delta g_{H_k} = \Delta g_{H_k} + \Delta g_{(i,j)} \cdot \frac{c_{i,j,k}}{\Delta A} \]

(3.6) \[ \Delta g_{H_k} = \Delta g_{H_k} \Delta A \]

B. Sum over all 2.5x 2.5 blocks (i,j)

For each block (i,j):

For each corner of the block:

(3.7) \[ x_1 = 111.19493 \cos \varphi_4 (\lambda_4 - \lambda_0) \]

(3.8) \[ y_1 = 111.19493 (\varphi_4 - \varphi_0) \]

(3.9) \[ x_2 = 111.19493 \cos \varphi_4 (\lambda_2 - \lambda_0) \]

(3.10) \[ y_2 = y_1 \]

(3.11) \[ x_3 = x_2 \]

(3.12) \[ y_3 = 111.19493 (\varphi_3 - \varphi_0) \]

(3.13) \[ x_4 = x_1 \]

(3.14) \[ y_4 = y_3 \]
For each elevation \((k)\), for each block \((i,j)\)

\[
(3.15) \quad D = \sqrt{x^2 + y^2 + H_k^2}
\]

\[
(3.16) \quad c_{i,j} = F(1) - F(2) + F(3) - F(4)
\]

\[
(3.17) \quad \text{where } F = \tan^{-1} \frac{xy}{HD}
\]

\[
(3.18) \quad \Delta g_{H_k} = \Delta g_{H_k} + c_{i,j} \Delta g_{i,j}
\]

The above complete the fundamental equations used for the upward continuation of anomalies.

3.2 Normal Gravity

The equations used for the computation of the vertical component of normal gravity are below. With the \(\varphi\), \(\lambda\), and \(H\) of the computation point

<table>
<thead>
<tr>
<th>Compute</th>
<th>Find</th>
</tr>
</thead>
<tbody>
<tr>
<td>(X = (N + H) \cos \varphi \cos \lambda)</td>
<td>(X)</td>
</tr>
<tr>
<td>(Y = (N + H) \cos \varphi \sin \lambda)</td>
<td>(Y)</td>
</tr>
<tr>
<td>(Z = (N(1 - e^2) + H) \sin \varphi)</td>
<td>(Z)</td>
</tr>
<tr>
<td>(N = \frac{s}{(1 - e^2 \sin^2 \varphi)^{1/2}})</td>
<td></td>
</tr>
<tr>
<td>(p^2 = x^2 + y^2)</td>
<td>(p^2)</td>
</tr>
<tr>
<td>(r^2 = p^2 + Z^2)</td>
<td>(r^2)</td>
</tr>
<tr>
<td>(K^2 = r^2 + c^2)</td>
<td>(K^2)</td>
</tr>
<tr>
<td>(h^2 = \sqrt{K^4 - 4p^2c^2})</td>
<td>(h^2)</td>
</tr>
<tr>
<td>(\sin^2 \alpha = \frac{K^2 - h^2}{2p^2})</td>
<td>(\alpha)</td>
</tr>
<tr>
<td>(\tan \beta = \frac{Z}{pcos \alpha})</td>
<td>(\beta)</td>
</tr>
</tbody>
</table>
\begin{align*}
(3.29) \quad q &= \frac{3}{2}[\alpha - 3 \cot \alpha (1 - \alpha \cot \alpha)] \\
(3.30) \quad q' &= \frac{3(1 - \alpha \cot \alpha)}{\sin \alpha} - 1 \\
(3.31) \quad w &= (1 - \sin^2 \alpha \cos^2 \beta)^{\frac{1}{2}} \\
(3.32) \quad \Gamma_\alpha &= \frac{KM \sin^2 \alpha}{2c_w} + \frac{w^2 a^2 q' \sin^2 \alpha}{2c_w} (\sin^2 \beta - \frac{1}{3}) \\
(3.33) \quad \Gamma_\beta &= \frac{w^2 a^2 \sin \alpha \sin \beta \cos \beta}{c_w} \\
(3.34) \quad \gamma_\alpha &= \Gamma_\alpha - \frac{w^2 c \cos^2 \beta}{w \tan \alpha} \\
(3.35) \quad \gamma_\beta &= -\Gamma_\beta + \frac{w^2 c \sin \beta \cos \beta}{w \sin \alpha} \\
(3.36) \quad \gamma_T &= \sqrt{\gamma_\alpha^2 + \gamma_\beta^2}
\end{align*}

where \( \gamma_T \) is the normal gravity in the vertical direction. Certain constants used in this program designated by \( c, KM, q, a, e \) refer to the International Ellipsoid and may be computed as described in the pertinent report.

4. Special Considerations

In the previous section we have tacitly assumed several procedures which are now appropriately discussed. The first question to be examined is why use 5'x 5' blocks, and why use 2.5x 2.5 blocks. Secondly, why use the integrated form of \( c_{i,j} \), only for the 2.5x 2.5 blocks, if at all.
4.1 Consideration of Block Size

The first question may be answered by considering two factors. The first is the area over which the integral summation is to be carried. In theory, the summation should be made over the entire sphere, or in the plane approximation over the entire plane. However, the effects of the distant area are small for the elevations (< 200km) of interest in this report. Moritz has estimated that it is sufficient to extend the summation over a radius equal to 10 times the height of the point. For a height of 10km, the radius would be 100km or approximately 1° in latitude. Areas of this size are most readily described in terms of 5'x 5' mean anomalies. For the higher elevations, the area to be considered would be larger and perhaps more easily represented by large size elements. However, this would complicate data preparations. It is thus felt that for elevations that would be considered for airborne measurements and similar encountered altitudes, 5'x 5' mean anomalies are sufficient. For the higher altitudes under consideration (100-200km), this size block is perhaps too small for the most economical computation, but they will satisfy the requirements.

The lowest elevation at which a computation may be made is limited by the size of the squares used in the integration. As the value of \( H \) becomes smaller, the size of the blocks in which in which the mean anomaly has been estimated must also be reduced in order to be able to adequately reflect the variations of the anomaly field. If the elevation was only \( dH > 3 \), the needed squares would be essentially point anomalies.

It is pertinent to discuss the elevation at which a certain size square will give satisfactory results. Various authors have made suggestions along these lines, either explicitly or implicitly. For example[2] Hirvonen has computed a template for the upward continuation. He concluded from his figures that 5'x 5' squares would not give the same accuracy as the template below 20km. On the other hand, the smallest radius of his template has a radius of 0.4\( H \), so that the radius for 20km = 8km. Now a circle of 8km would be composed of approximately 4 5'x 5' squares, which if the \( c_{i,j} \) are calculated in the integrated form, should yield better accuracy than the template. If we use the criteria of 0.4\( H \) the lowest elevation corresponding to the accuracy of the template method would be:

\[
H = \frac{a}{2(.4)} = \frac{a}{.8}
\]
where a is the side of the square. Taking for a 5'x 5' block, we have a = 8km so that H = 10km. In words we might conclude that the size of the square needed is approximately 0.8 of the point elevation.

Moritz [3] recommends another procedure when dealing with square elements. This involves evaluating the upward continued anomalies at the center of adjacent 5'x 5' squares surrounding the center square in which the computation point lies. Interpolation for the given point is then carried out from the 3 given values. This procedure is said to be necessary for elevations smaller than 70 km. Even though an interpolation is performed in this method, it still should not readily improve the situation for the lower elevation (~10 km).

In addition to the above described method, Moritz also described a method in which the effects of gradients within a 5'x 5' square may be taken into account for the non-vertical components for elevations less than 10km.

Orlin has also mentioned this problem of square size. He adopts a figure giving the ratio of distance to block size as 2 to 1. Thus for an elevation of 16 km a square size of 8 km (5') would be needed. However, these figures were only used to within 10' of the computation point, with a template procedure being applied for that inner zone.

We might formulate the problem in a slightly different way. Moritz [4] has given an expression for the expected accuracy of the upward continued anomaly based on a rectangular block with side a and b containing the mean anomaly. This is:

$$m^2_H = \frac{1}{H^2} \frac{A}{8\pi} m^2$$

where A is the area (ab) of the block and $m^2$ is the expected standard error of the estimated mean anomaly in the square. $m^2$ may be taken as:

$$m^2 = \frac{\alpha^2}{6} (a^2 + b^2)$$
where $\alpha^2$ is a constant depending on the covariances of the anomalies. Using this we can write:

$$\frac{m}{H} = \frac{\alpha^2}{48\pi} \frac{(a^2 + b^2)}{H^2} ab$$

Now suppose that we ask the following question:

What is the size of a block $(a_2, b_2)$ needed at a certain elevation $H_2$, to yield the same accuracy at an elevation $H_1$ with a block size $a_1$, $b_1$?

We answer this by taking the ratio of the respective standard errors:

$$\frac{\frac{m}{H_2}}{\frac{m}{H_1}} = \frac{\frac{a_2}{H_2^2}}{\frac{a_1}{H_1^2}} \frac{(a_2 + b_2)}{a_2 b_2}$$

If we approximate the rectangular blocks with squares, $a = b$, we have:

$$\frac{\frac{m}{H_2}}{\frac{m}{H_1}} = \frac{\frac{a}{H_2^2}}{\frac{a}{H_1^2}}$$

If the accuracies at these elevations are to be the same, this ratio must be 1, so we have

$$a_2 = a_1 \sqrt{\frac{H_2}{H_1}}$$

Let us accept the value of $a_1$ as 8km (5' sq) and compute the value of $a_2$ corresponding to selected values of $H_1$ and $H_2$. These results are presented in Table 1.
Table 1

Value of $a_2$ for given $H_1$ and $H_2$

<table>
<thead>
<tr>
<th>$H_2$</th>
<th>$H_1$</th>
<th>20</th>
<th>15</th>
<th>10</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>6.9</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>5.7</td>
<td>6.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>4.0</td>
<td>4.6</td>
<td>5.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>3.6</td>
<td>4.1</td>
<td>5.1</td>
<td>7.2</td>
<td></td>
</tr>
</tbody>
</table>

If we accept adequate accuracy from 5' square at $H = 20$ km, we would expect a similar accuracy at 5 km with a 4 km (2.5) square. Other conclusions follow similarly.

It is apparent from the figures that there is some disagreement on what size of block should be used for the lower elevations. On the other hand, whether we use the rule of $0.8H$ for $a_2$ or the $a_2$ from Table 1 it appears that the use of 2.5 squares is valid for elevations down to 5 km and perhaps 4 km. Tests pertinent to these conclusions are described in the next section.

4.2 Integration Considerations

We have discussed two forms of $c_{ij}$, a non-integrated form and the integrated form. The question arises where and with what size squares should the particular forms be used. Granting that it is sufficient to use 5' x 5' and 2.5 x 2.5 blocks, is it sufficient to use the integrated form of $c_{ij}$ for the 2.5 blocks, and the non-integrated form for the 5' x 5' blocks?

To answer the above question a test situation was examined under various conditions. A 3° (φ) x 5° (λ) area was selected with the coordinates:

$38^\circ \rightarrow 35^\circ$

$240^\circ \rightarrow 245^\circ \ (115^\circ \ - \ 120^\circ)$

A set of 5' x 5' mean free-air anomalies was established for this
area from estimated Bouguer anomalies and mean elevations. Near the center of this area a group of stations was selected for these test computations. The location of these points is described below:

![Diagram](image)

Though points 1, 2, and 3 are of primary interest, the actual coordinates for all the points are now given:

<table>
<thead>
<tr>
<th>Point</th>
<th>Latitude</th>
<th>Longitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>36° 32' 30&quot;</td>
<td>242° 32' 30&quot;, -117° 27' 30&quot; W</td>
</tr>
<tr>
<td>2</td>
<td>36 33 45</td>
<td>242 33 45, -117 26 15 W</td>
</tr>
<tr>
<td>3</td>
<td>36 34 00</td>
<td>242 34 00, -117 26 00 W</td>
</tr>
<tr>
<td>4</td>
<td>36 32 30</td>
<td>242 27 30, -117 32 30 W</td>
</tr>
<tr>
<td>5</td>
<td>36 37 30</td>
<td>242 32 30, -117 27 30 W</td>
</tr>
<tr>
<td>6</td>
<td>36 32 30</td>
<td>242 37 30, -117 22 30 W</td>
</tr>
<tr>
<td>7</td>
<td>36 27 30</td>
<td>242 32 30, -117 27 30 W</td>
</tr>
</tbody>
</table>
Points 1, 4, 5, 6 and 7 lie at the center of 5'x 5' squares. Point 2 is located at the center of a 2'5x 2'5 square while point 3 has no special location, except lying within that same 5' block as points 1 and 2.

Points 5 and 6 are needed for an interpolation (with point 1) for finding \( \Delta g_H \) from the so-called sub-station. The method for interpolation is described by Moritz and basically is the fitting of a plane through points 1, 5 and 6 which lie at 5'x 5' centers and interpolating for the gravity anomaly at points 2 and 3. If we consider a coordinate system with the origins at point one and computing
\[
 x = 6371 \cos \varphi_o (\lambda - \lambda_o) \quad \text{and} \quad y = 6371 (\varphi - \varphi_o)
\]
where \( \varphi_o, \lambda_o \) are the coordinates of point one, we can form an expression for the \( \Delta g_H \) using equation 5 of Moritz [5]. We find:

for point 2
\[
\Delta g_{H2} = \frac{\Delta g_1}{2} + \frac{\Delta g_5 + \Delta g_6}{4}
\]

for point 3
\[
\Delta g_{H3} = 0.40 \Delta g_1 + (\Delta g_5 + \Delta g_6) \cdot 0.3
\]

The anomalies around the three points of interest (1, 2, 3) were chosen in three different forms:

1. the original 5'x 5' mean free-air anomalies
2. the 2'5x 2'5 mean anomalies with the same mean in this square as in the corresponding 5' square
3. the 2'5x 2'5 mean anomalies with various values in them, computed such that their mean would be the same in the corresponding 5'x 5' square.

The situation as described in the best form is shown in Figure I.
The situation as described in the second form is identical with that of Figure I except within each square are four values, the same values as in Figure I representing the 2.5 mean anomalies.

The situation as described in the third form is shown in Figure II.

<table>
<thead>
<tr>
<th>36° 40'</th>
<th>36° 35'</th>
<th>36° 30'</th>
<th>36° 25'</th>
</tr>
</thead>
<tbody>
<tr>
<td>-16.0</td>
<td>20.0</td>
<td>39.6</td>
<td>40.0</td>
</tr>
<tr>
<td>40.0</td>
<td>50.0</td>
<td>60.0</td>
<td>30.0</td>
</tr>
<tr>
<td>96.2</td>
<td>73.0</td>
<td>74.0</td>
<td>85.0</td>
</tr>
<tr>
<td>47.0</td>
<td>69.0</td>
<td>91.4</td>
<td>40.0</td>
</tr>
<tr>
<td>13.2</td>
<td>26.2</td>
<td>-42.1</td>
<td>10.0</td>
</tr>
<tr>
<td>-4.7</td>
<td>-25.1</td>
<td>-40.1</td>
<td>-67.4</td>
</tr>
</tbody>
</table>

2.5x 2.5 Mean Anomalies in Test Case

19.
The values of $\Delta \sigma_{ii}$ were computed at 10 elevations using 6 different methods or anomalies. These methods are now described:

A - using only 5' x 5' means, $c_{ij}$ not integrated

B - using only 5' x 5' means, $c_{ij}$ integrated

C - using only 5' x 5' means, $c_{ij}$ integrated, values interpolated from substation calculation using surface fitting

D - using 5' x 5' means, $c_{ij}$ integrated and with 2.5' x 2.5' means with $c_{ij}$ integrated, with the same mean in the 2.5' square as in the corresponding 5' square.

E - using 5' x 5' means with $c_{ij}$ non-integrated, and with 2.5' x 2.5' means with $c_{ij}$ integrated, with the same mean in the 2.5' square as in the corresponding 5' square.

F - using 5' x 5' means with $c_{ij}$ non-integrated, and with 2.5' x 2.5' means with $c_{ij}$ integrated, with various mean values (Fig. II) but with the same overall mean as the 5' square.

These results for these computations are presented in Table 1 where the evaluations at points 1, 2, and 3 were made. There are several conclusions that may be drawn from this table. These will be drawn for the lowest elevation as the differences between the methods decreases with elevation, as does the anomaly itself.

For cases A through E, the method that should be regarded as best would be case D. Method B and Method D should give identical results since the same mean anomalies have been used in the two square sizes. It is apparent from Table 1 that Method A gives incorrect results when the point is at the center of the 5' square and fair results otherwise. The smallest difference between the two methods is .4 mgals at the highest elevation while the elevation at which the deviation does not exceed 1 mgal is near 12 km. Method C yields results consistently lower than the accepted value found in Method D. At points 2 and 3 an error of 4 mgals is found for the lowest elevation, while the result at 16 km shows an error of 0.5 mgal. Even greater discrepancies are apparent when compared with the results from Method F, but the real comparison of interest is with Method D. It appears that this method of interpolation
yields erroneous results in this example and there is no reason to believe it would not hold for other situations. Besides this fact, additional work is required for the computation of the anomalies at the substations which can substantially increase the needed computer time.

Method E enables us to see if it is possible to use only the integrated $c_{ij}$ for the $2\frac{2}{3}x 2\frac{1}{2}$ means while using the direct form for the 5' squares. The answer appears to be yes, as the maximum difference between E and D was 0.1 mgals.

It is of interest to compare Method F, where rather erratic anomalies for the 2:5 blocks have been used, with Method D. If these anomalies had actually existed as error of using D over F would be 2.4, 3.5, and 3.8 mgals for points 1, 2, and 3 respectively at $H = \frac{4}{1}$ km. The elevation at which the difference between D and F is near 1 mgal is at 12 km for the three points. We could than say that below 12 km we should use smaller size squares if an accuracy of integration below 1 mgal is to be maintained. From Table 1 (or the equation for $a_0$) we could than conclude that the 2:5x 2:5 blocks may be used down to an elevation of $4 \frac{1}{2}$ km.

In summary, we may tentatively reach the following conclusions:

1. It is necessary to use the integrated form of $c_{ij}$ only for the closest squares which are the 2:5x 2:5 blocks, with the un-integrated form of $c_{ij}$ being used for the farther 5'x 5' blocks.

2. If no 2:5x 2:5 blocks are used, the non-integrated form of $c_{ij}$ for the 5' means should be used no lower than 12 km.

3. If no 2:5x 2:5 blocks are used the integrated form of $c_{ij}$ for the 5' means should be used no lower than 12 km.

4. The substations computation and subsequent surface interpolation does not give correct results for the lower elevations.

Points 2 and 3 need comment as taken together they look trivial. However, the reasoning behind each is different. Statement 2 arises from the inaccuracy in the non-integrated $c_{ij}$ over the $c_{ij}$ in the integrated form. Statement 3 arises from an expected variation in the
anomaly field in the surrounding 215x 215 blocks. It just happens that both considerations yield the same elevation criteria. In fact, if we compare Method A with Method F, it might appear that for elevations above 8 km, the two methods yield results within 1 mgal. This result is fortuitous at best, and no conclusions may be drawn from such comparison.
Table 1
Comparison of Upward Continued Anomalies

<table>
<thead>
<tr>
<th>Elev.</th>
<th>(1) 36° 32' 30&quot; -117°27' 30&quot;</th>
<th>(2) 36° 33' 45&quot; -117° 26' 15&quot;</th>
<th>(3) 36° 34' 00&quot; -117° 26' 00&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>(km)</td>
<td>A</td>
<td>B</td>
<td>D</td>
</tr>
<tr>
<td>4</td>
<td>58.8</td>
<td>35.1</td>
<td>35.1</td>
</tr>
<tr>
<td>5</td>
<td>41.3</td>
<td>29.4</td>
<td>29.4</td>
</tr>
<tr>
<td>6</td>
<td>31.5</td>
<td>24.8</td>
<td>24.8</td>
</tr>
<tr>
<td>7</td>
<td>25.2</td>
<td>21.1</td>
<td>21.1</td>
</tr>
<tr>
<td>8</td>
<td>20.8</td>
<td>18.1</td>
<td>18.1</td>
</tr>
<tr>
<td>9</td>
<td>17.4</td>
<td>15.5</td>
<td>15.5</td>
</tr>
<tr>
<td>10</td>
<td>14.8</td>
<td>13.4</td>
<td>13.4</td>
</tr>
<tr>
<td>12</td>
<td>11.0</td>
<td>10.2</td>
<td>10.2</td>
</tr>
<tr>
<td>14</td>
<td>8.3</td>
<td>7.8</td>
<td>7.8</td>
</tr>
<tr>
<td>16</td>
<td>6.5</td>
<td>6.1</td>
<td>6.1</td>
</tr>
</tbody>
</table>
4.3 Accuracy of the Upward Continued Anomaly

It is of interest to know to what degree of reliability we can attach to the upward continued anomaly. In sections 4.1 and 4.2 two considerations have been made with respect to the subject: block size and integration requirements within a block. Of more pertinent interest is to consider the influence of the inaccuracy of the used gravity material and to question how far away from the computation point need the summation be carried out. Moritz [3,4] has extensively analyzed these problems so that we take from his work some pertinent equations.

4.3.1 Statistical Accuracy

In a previous report [7], the author outlined the following steps:

If the error covariance function of gravity anomalies is taken as,

\[ \sigma(s) = \sigma_0 e^{-c^2s^2} \]  

(7, equation numbers refer to the Moritz [4] report)

where \( \sigma_0, c^2 \) are constants and \( s \) is the anomaly separation. Moritz then shows that the standard error of the upward continued anomaly is,

\[ m_H^2 = \frac{1}{8H^2} \frac{\sigma_0}{c^2} \]  

(18)

Consider next the error of representation in anomaly block with sides \( a \) and \( b \). Now is a point anomaly in this block is taken to represent the mean anomaly in the whole block, this error of representation is:

\[ m^2 = \frac{\alpha^2}{6} (a^2 + b^2) \]  

(42)

where \( \alpha^2 \) is a constant found in the expression:

\[ C(s) = C_0 - \alpha^2s^2 \]  

(40)
\[ \sigma^2 = C_0/d^2 \text{ where } d \text{ is the separation at which } C(r) \text{ equals } C_0/2. \text{ C(r) is the covariance of the gravity anomalies and } C_0 \text{ is the variance of the anomalies. Moritz shows that:} \]

\[ m_H^2 = \frac{1}{H^2} \frac{A}{8\pi} m^2 \quad (48) \]

so that taking \( A = ab \), the area of the square and \( m \) from (42) we have:

\[ m_H^2 = \frac{1}{H^2} \frac{\sigma^2}{48\pi} \]

This equation should be evaluated for the given circumstances and area of uplifting. Some previous calculations show one estimate for this error using 5'x 5' squares could be:

\[ m_H = \pm \frac{5}{H} \text{ mgals.} \]

Moritz obtained a similar but slightly smaller value.

4.32 Accuracy from Individual Block Accuracies

We have written the upward continuation formula in the form:

\[ \Delta g_H = \frac{H}{2\pi} \sum \sum \Delta g \frac{dx \, dy}{D_0^3} \]

This equation can be regarded as simply a function composed of a summation of a certain number of elements. If we say that we know the accuracy of each of the anomalies (\( m_{\Delta g} \)) to be used in the summation, and neglecting the correlations between adjoining blocks due to non-independent estimation, we may write:

\[ m_{\Delta g_H}^2 = \frac{H^2}{(4\pi)^2} \sum \sum \left( \frac{dx \, dy}{D_0^3} \right)^2 m_{\Delta g}^2 \]

or in terms of the coefficient \( c \)

\[ m_{\Delta g_H}^2 = \frac{1}{4\pi^2} \sum \sum c^2 m_{\Delta g}^2. \]
This equation may be used in the computer program to be described simply by using the values of \( m \Delta g \) instead of \( \Delta g \) as input, and modifying several statements so that \( c^2 m^2 \Delta g \) is computed instead of \( c \Delta g \).

4.33 Maximum Accuracy

It has been previously mentioned that the original formulation of the upward continuation problem requires a summation over the whole earth sphere. When the problem is transferred to the plane the corresponding summation is over the entire plane. In practice it is not necessary to do such a complete summation since the values of \( c \) become smaller for distant areas. Moritz [3] has considered the question and his answer is that it is sufficient to go out to 10 times the elevation in order to be assured of an accuracy of \( \pm 1 \) mgal. This figure assumes a mean anomaly of 10 mgals beyond the integration area. If this mean is smaller than specified, the accuracy of the upward continuation will decrease in direct proportions.

To be on the pessimistic side, though, we can accept the fact that if we go to a distance of 10 times the elevation the error committed would be no larger than \( \pm 1 \) mgal. In some practical tests that have been made, we have added or deleted up to 30' blocks of anomalies beyond 10 times the elevation and found effects of only 0.1 or 0.2 mgals, thus leading to the belief that the figure of \( \pm 1 \) mgal is somewhat pessimistic.
5. General Program Description and Flow Diagram

5.1 General Program Description

The computer program designed to perform the upward continuation of gravity anomalies is basically set up in two components. The first component basically reads in from cards the 5'x 5' mean anomalies in a block 30' wide in latitude extent and as long as needed (in longitude). This block is brought in and stored in a two dimensional array. After all cards in a block are stored, the anomalies are written on tape one row at a time (longitude extent). For each row, the latitude (deg. min.) longitude (deg., min., + east) and the number of anomalies are written. After each row the next row is written until all 5' strips in the 30' block are written. The program then returns for a completely new block and storage pattern. This continues until the supply of blocks is exhausted. This created tape (#9) is now rewound and is ready for use in calling the main body of the program which is in a subroutine: SUBS43.

The first part of SUBS43 reads all the data from the tape (#9) created in the first part of the program. All the anomalies are stored with information pertaining to the location of the first anomaly in the string and the number of anomalies. The advantage of such a method is that the location of each anomaly need not be specified. These 5'x 5' anomalies are stored in the array ANOM1.

The machine is next informed if it is dealing with a latitude (999) or a longitude (9999) profile. This feature is described in a later section, but briefly it is a means to make computations and data handling more efficient by identifying a predominate north-south (999) or east-west (9999) profile. Now a card is read that contains four 2'1/2x2'1/2 mean anomalies and the coordinates of the NW corner of the 5' square. This information is stored in the appropriate location in the array ANOM2. Consecutive cards having the same latitude (or longitude) are read in until the end of a strip is reached. Then a new strip is started and finished. This process is repeated until all the 2'1/2 anomalies are in core. In addition to the proper storage of the anomalies, an additional computation is done to zero the 5'x 5' anomaly which is being duplicated by the 2'1/2 information. In this manner, it is possible to combine both types of anomalies in the needed summation.
The computation then proceeds to read-in the location of a point at which the upward continued anomaly is to be computed. The location is specified by the latitude, longitude, number of elevations, and the sequence of elevations at which the anomaly is to be obtained. Thus it is possible to compute at several (up to 15 at present) elevations. This feature has the advantage of reducing the total amount of computation for the several elevations over the separate program submission for each elevation. In practice this means that little additional computer time is necessary for more than one elevation.

Having the coordinates, the first computation is the summation of the upward continued equation over the 5'x 5' mean anomalies. The next computation is the summation over the $2\frac{1}{2} \times 2\frac{1}{2}$ means using the integrated form of $c_{ij}$. Where the summation and computation takes place depends on whether we are dealing with an east-west or north-south profile. Before the print out of the final answers, the remaining step is to compute the normal gravity at the given latitude and elevation (regarded as above the reference ellipsoid). Finally the answers are given for the point: the latitude, longitude, elevations, with the anomaly and normal gravity for each elevation is printed out. All computations being completed for this point, the program returns for another point, repeating all computation until the point supply is exhausted.

5.2 The Flow Diagram

In order to show in a schematic way the operation of this program a flow diagram has been made and is given below. The symbols used in this diagram generally correspond to the ones that are found in the program. The flow diagram was designed by Doug Fleckner of the OSU Computation Center.
SUBROUTINE SUB543 (NBL, ITAP, KTAP)

CÖN = III.19493
DDEG = 0.0416667
DLMD = 0.083333

IDIM = 6 * NBL
IVAR = 6

ZERO ANÖM 2
ARRAY

K = 1
K ≤ IDIM
K = K + 1

READ ANOMALIES FOR
STRIP K FROM TAPE
INTO ANÖM1

KK1 = K

GGLAT = NORTH-MÖST LAT
GGNG = WEST-MÖST LÖNG
OF ANÖMALIES

READ TYPE OF
2½ x 2½ STRIPS

LATITU = 1

ITYE = 999?
YES
LATITU = 0
NO

DIMENSIÖN
TEMP(4), H(15), DG(15),
F1(15), F2(15), F3(15), F4(15),
ANÖM1(100,48)
ANÖM2(100,10)
NUMBER(6), III(6), JJJ(6)
\[ x = \text{CONST} \times (\text{FLMDI} - \text{FLMDO}) \]
\[ = 1.943 \times \cos(\phi_{ij}) \times (\text{FLMDI} - \text{FLMDO}) \]

\[ x_{SQ} = x^2 \]

\[ \begin{cases} I = 1 \\ I \leq NHS? \\ I = I + 1 \end{cases} \]

\[ \begin{cases} \text{NO} \\ \text{YES} \end{cases} \]

\[ \text{FLMDI} = \text{FLMDI} + 5 \]

\[ d_1 = \sqrt{x_{SQ} + y_{SQ} + \text{H}(i)^3} \]

\[ c = \frac{\cos(\phi_{ij}) \times \text{H}(i)}{(d_1)^3} \]

\[ \text{DG}(i) = \text{DG}(i) + c \times \text{ANOM1}(j, k) \]
I = 1
I ≤ NHI?
I = I + 1

H5Q = \( (H(I))^2 \)

D2 = \( \sqrt{X1SQ + Y1SQ + H5Q} \)
D3 = \( \sqrt{X1SQ + Y2SQ + H5Q} \)

F2(I) = \( \tan^{-1}\left(\frac{X1 \cdot Y1}{H(I) \cdot D2}\right) \)
F3(I) = \( \tan^{-1}\left(\frac{X1 \cdot Y2}{H(I) \cdot D3}\right) \)

C = F1(I) - F2(I) + F3(I) - F4(I)

DG(I) = D6(I) + C \times \text{ANOM2 (J, K)}

F4(I) = F3(I)
F1(I) = F2(I)

B

K = 1
K ≤ KK?
K = K + 1

NO

CALCULATE N-WMOST COORDINATE OF Kth LATITUDE STRIP PHI, FLMDI

NUM = No. OF 2\(\frac{1}{2}\)x2\(\frac{1}{2}\) SQUARE IN STRIP

TEMP1 = 111.19493 \times (FLMDI - FLMDO)
TEMP2 = 111.19493 \times (FLMDI + 2\(\frac{1}{2}\) - FLMDO)
CONST = 111.19493

COSP = COS (PHII)

X = TEMP1 \times COSP
= 111.19493 \times (FLMDI - FLMDO) \times COS(PHI)
X1 = TEMP2 \times COSP
= 111.19493 \times (FLMDI + 2\(\frac{1}{2}\) - FLMDO)

XSQ = X^2
X1SQ = (X1)^2
C

WRITE LAT., LONG.

I = 1
I ≤ NHS
I = I + 1

D

NO

DG(I) = DG(I) * 0.1591549

CALL SUBROUTINE (T) = SGAMMT ( )

YES

WRITE H(I), DG(I), T

PUNCH LAT., LONG., DG(I), T, H(I)

36.
6. The FORTRAN Program

6.1 Preliminary Comments

Before the specific input formats of the program are discussed, it is appropriate to describe a few preliminary points. The program is designed to accept two kinds of cards containing mean free-air anomalies. These cards will contain 5'x 5' and 215x 215 means. The 5' cards are handled in blocks; the latitude extent is 30' and the longitude extent is the complete extent of coverage desired for that latitude range. Each block consists of a sequence of cards that contain (on each card) 6 anomaly values along 30 minutes of latitude at a constant longitude. Each anomaly value then represents the anomaly in a 5'x 5' square whose longitude is specified on the card, and whose latitude may be computed from the latitude punched on the card. The coordinates of the square refer to the northwest corner of the square with the longitude being considered from 0° to 360° (positive east).

If a block is 5° wide, there will be 5 x 12 = 60 cards with a total of 60 x 6 = 360 anomalies. Figure III represents this picture:

---

Figure III
5'x 5' Blocks
The format of each 5' card is (6F5.1, 28X, 2(I3, I2)) with the 6 anomalies punched in the first 30 columns (5 cols/anomaly) and with the latitude and longitude (positive east) of the NW corner of the 30' strip appearing in columns 59-68. Specifically we have:

<table>
<thead>
<tr>
<th>col.</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-5</td>
<td>anomaly 1</td>
</tr>
<tr>
<td>6-10</td>
<td></td>
</tr>
<tr>
<td>11-15</td>
<td></td>
</tr>
<tr>
<td>16-20</td>
<td></td>
</tr>
<tr>
<td>21-25</td>
<td></td>
</tr>
<tr>
<td>26-30</td>
<td></td>
</tr>
<tr>
<td>59-61</td>
<td>latitude (deg)</td>
</tr>
<tr>
<td>62-63</td>
<td>latitude (min)</td>
</tr>
<tr>
<td>64-66</td>
<td>longitude (deg) $0^\circ-360^\circ$ (+ east)</td>
</tr>
<tr>
<td>67-68</td>
<td>longitude (min)</td>
</tr>
</tbody>
</table>

The number of 5'x 5' blocks to be used is particularly dependent on the elevations. As previously stated, the anomaly coverage should extend to a distance approximately 10 times the elevations from the computation point or profile. If a profile is used the 5' blocks should be chosen to border the profile as shown in Figure IV where an east-west profile is shown. Since the 5' blocks are 30' in latitude extent it is necessary at times to include more anomalies than actually needed but the only effect is a slightly increased computer running time.
Figure IV

Example of Profile and 5'x 5' Block Relation

Similar examples could be shown for north-south and oblique profiles. With oblique profiles, the 5' blocks would not need to be chosen in a square fashion but may be set up in a stepped way as shown below.

Figure V
In addition to the 5' x 5' mean anomalies that are used in the 5' blocks, the program also used the 2.5' x 2.5' mean anomalies that are especially important for low elevations (on the order of 7 or 8 km). These anomalies are needed only in the immediate vicinity of a computation point or profile. In order to define immediate vicinity we consider two cases. First consider an example when the computation point is situated at the center of a 5' x 5' square. In this case, 2.5' x 2.5' anomalies are needed in the 5' square in which the point is situated and in the 8 adjacent 5' x 5' squares. This is shown in Figure VI.

![Figure VI](image)

**Figure VI**

2.5' x 2.5' Anomalies

Surrounding Computation Point

A similar example may be set up for the needs of 2.5' mean anomalies along a profile. If we consider a profile along an integer 5' of latitude or longitude it is sufficient to use the 2.5' anomalies to 5' on each side of the anomaly and 5' at the ends. This is shown in Figure VII.
Figure VII
Location of 2:5 Mean Anomalies on Profiles

Notice that along a profile, the computation point is taken at the center edge of a 5' x 5' square. This is usually done for symmetry purposes and is not necessary for elevations above 10 km, if at all.

The 2:5 x 2:5 mean anomalies are punched four to a card representing the four 2:5 means in a 5' x 5' square. The first four fields on the card are the four mean anomalies in order 1, 2, 3, 4 with the latitude and longitude of the northwest corner of the 5' square being specified after the four mean anomalies. The four anomalies are punched in the first 24 columns, 6 cols/anomaly, with the latitude and longitude punched in columns 59-68. The card format is: ([4F6.1, 34X, 2(I3, I2)]. The order of the anomalies punched on the card is shown in Figure VIII.
Figure VIII

Arrangement of 2:5
Means for Punching
Within 5' Square

Specifically the format for the 2:5x2:5 mean anomalies is:

<table>
<thead>
<tr>
<th>col.</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-6</td>
<td>anomaly 1</td>
</tr>
<tr>
<td>7-12</td>
<td>&quot; 2</td>
</tr>
<tr>
<td>13-18</td>
<td>&quot; 3</td>
</tr>
<tr>
<td>19-24</td>
<td>&quot; 4</td>
</tr>
<tr>
<td>59-61</td>
<td>latitude (deg)  (- south)</td>
</tr>
<tr>
<td>62-63</td>
<td>latitude (min)</td>
</tr>
<tr>
<td>64-66</td>
<td>longitude (deg)  0° to 360° (+ east)</td>
</tr>
<tr>
<td>67-68</td>
<td>longitude (min)</td>
</tr>
</tbody>
</table>

For more efficient computer usage, it is necessary to specify a type of profile that is being computed. The profile type may be referred to as an east-west (or longitude) profile and a north-south (or latitude) profile. If an oblique profile is being used it is only necessary to describe it in its predominant sense. That is, it is predominantly east-west or predominantly north-south. If no predominant sense can be attached it is unimportant which type of profile it is called, but future input arrangements must be consistent with the
profile types. Additionally, if only a small group of points (or just one point) is being computed, either east-west or north-south profile designations may be used, with the input arrangements of the 2:5 means in agreement with the profile type to be described below.

Consider first the case of an east-west (longitude) profile which will be identified in the program by 9999. For this type, (9999) the 2:5x2:5 cards should be read as follows: The first 2:5 card represents the northwest corner of the strip. The second and succeeding cards at that latitude are read in order of increasing longitude (positive east) to the last card needed at that latitude. After this card comes the start of a new latitude, 5' minutes below the preceding one. Though the starting longitudes are usually the same for the 2:5 cards they need not be so. Again this set of cards is used to fill out a row of constant latitude along a varying longitude (increased by 5' for each card). If the profile is east-west, only two rows of cards with 2:5 anomalies are needed. For oblique profiles more rows (i.e. different latitudes) with varying starting and ending longitude will be used. To summarize, the 2:5 cards are used in order of highest latitude, and longitude first (NW corner of profile) followed by increasing the longitude and the starting latitude, following to the end of the planned profile, decrementing the latitude by 5' and starting again from the origin longitude. Further clarification is given in Figure IX.

![Diagram of profile loading](image-url)

**Figure IX**
Example of Loading of 2:5 Anomalies for East-West (9999) Profiles

43.
A similar procedure is used for reading in the 215 mean anomalies for north-south (indicated by 999) profiles. The first card is for the NW square used, followed by the cards for the same longitude, but with the latitude decreasing by 5' for each card. After a column is filled up (i.e., to cover areas needed for the profile), the column starting at the north edge is started at a longitude increased 5' from the preceding. This is continued until the desired anomalies are read in. For the usual NS strip there will only be 2 vertical strips of cards. Further clarification is given in Figure X.

Figure X
Example of Loading of 215 Anomalies for North-South (999) Profiles
6.2 Detailed Input Formats

In the previous sections we have outlined some of the steps and principles to be followed in the preparations of data for this program. Now it is necessary to become more specific. First we write down the information needed in a general form:

1. number of 30' 5'x 5' anomaly strips (NBL)

2. latitude, longitude of 30' strip
   5' cards in above strip
   (repeated for all necessary strips)

3. type of profile (999 or 9999)

4. needed 215x 215 mean anomalies

5. points and elevation at which the computation is to be made.

Specifically:

1. number of 30' 5'x 5' anomaly strips (NBL) format (I2)
   (col 1-2 = NBL)

For each 30' 5'x 5' block we have the following:

2. NW corner of 30' 5'x 5' block

   col:
   1-2   NW corner of strip (latitude - deg)
   4-5   "      "      "      (latitude - min)
   7-9   "      "      "      (longitude - deg, + east (0° to 360°))
   11-12 "      "      "      (longitude - min)

The format for this card is : (I2, I3, I4, I3)
For the above specified strip, all 5'x 5' cards are entered.
The format for this card is: (6F5.1, 28X, 2(I3, I2))

45.
col:  
1-5  anomaly 1
6-10  "  2
11-15  "  3
16-20  "  4
21-25  "  5
26-30  "  6
59-61  latitude (deg)  (- south)  
62-63  latitude (min)  
64-66  longitude (deg, + east (0° to 360°))  
67-68  longitude (min)  

This format has been described in detail in section 6.1. The last card in 30' 5'x 5' strip: 99 in col. 60.61. Step 2 is repeated for all MBL (specified in l) blocks.

3. ITYPE =  
999 for predominantly north-south profile  
9999 for predominantly east-west profile  
see text for further discussion

col 1-4, format H4

4. 215x 215 mean anomalies  
each card contains 4 anomalies in a 5'x 5' square.

col.  
1-6  anomaly 1  
7-12  "  2  
13-18  "  3  
19-24  "  4  
59-61  latitude (deg)  (- south,  
62-63  latitude (min)  
64-66  longitude (deg)  0° to 360° (+ east)  
67-68  longitude (min)  

see Figure VIII  

of NW corner of 5' block
If ITYP = 999 (north-south strip) these cards are arranged in decreasing latitude (northern latitude first) within a given longitude, and in increasing (+ east) longitude (western most longitude first).

If ITYP = 9999 (east-west strip) these cards are arranged in increasing longitude (western most longitude first) within a given latitude, and in decreasing latitude (northern latitude first).

The last 215x215 card is indicated by a 99 in col 60-61.

5. Computation point information:

latitude (deg, min, secs), longitude (deg, min secs),
number of elevations, elevations at which upward continuation is to take place.
for each point:
card a:

<table>
<thead>
<tr>
<th>col:</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2</td>
<td>lat (deg)</td>
</tr>
<tr>
<td>4-5</td>
<td>lat (min)</td>
</tr>
<tr>
<td>7-11</td>
<td>lat (secs)</td>
</tr>
<tr>
<td>12</td>
<td>long (sign) (+ or -)</td>
</tr>
<tr>
<td>13-15</td>
<td>long (deg) (+ east or - west)</td>
</tr>
<tr>
<td>17-18</td>
<td>long (min)</td>
</tr>
<tr>
<td>20-24</td>
<td>long (secs)</td>
</tr>
<tr>
<td>26-27</td>
<td>number of elevations</td>
</tr>
<tr>
<td>29-36</td>
<td>elevation 1</td>
</tr>
<tr>
<td>37-44</td>
<td>&quot; 2</td>
</tr>
<tr>
<td>45-52</td>
<td>&quot; 3</td>
</tr>
<tr>
<td>53-60</td>
<td>&quot; 4</td>
</tr>
<tr>
<td>61-68</td>
<td>&quot; 5</td>
</tr>
</tbody>
</table>

47.
card b (if needed) remaining elevations (5 to a card)
card c (if needed) " " (" " " )

The format for card a is, (I2, I3, F6.2, A1, 2I3, F6.2, I3, 1X, 5F8.4). The format for card b is 5F8.4.

The above information is repeated for all points to be used in the computation. The end of the run is signified by a card with a 99 in col 1-2.

6.21 Special Input Arrangements

There are two special cases that might arise in the use of the upward continuation program that could be of interest. They both pertain to the situation when we do not have any 2:5x 2:5 mean anomalies.

The first case is when elevations above 12 (or more safely 20 km) are being considered. Thus we have seen that it is sufficient to consider the non-integrated form of the c_{ij}. Our input would then consist of the following:

1. sequence of 30' 5'x 5' anomaly strip
2. type of profile (either 999 or 9999, it is immaterial)
3. card with 99 in col. 60-61
4. computation point information

Note that no 2:5x 2:5 mean anomalies are used.

The second case arises when you are below 12 km (or more safely 20 km) where the c_{ij} in the integrated form must be considered. Without using the 2:5x 2:5 actual anomalies we can safely go down to 10 km. This is done by setting up a 2:5x 2:5 card with the same mean in each square as in the 5'x 5' square. These anomalies are used in the pertinent area. The input would look the same as the general case.
6.3 Output

In general the printed output consists of the following:

1. The original 5'x 5' mean anomalies in 5' strips in an east-west direction.

2. The profile designation

3. The 5'x 5' mean anomalies in 5' strips in an east-west direction in which the pertinent 5' squares have been zeroed due to having been replaced by 2:5x 2:5 mean anomalies.

4. The 2:5x 2:5 mean anomalies in a form identifiable with the type profile designated.

5. The results under appropriate headings giving the latitude, longitude, and for all input elevation, the upward continued anomaly and the normal gravity corresponding to that latitude and elevation.

In more detail:

1. For every original 5'x 5' strip the following is printed:
   latitude, longitude, and number of anomalies in that strip.

The latitude and longitude refer to the northwest corner of the strip. This information is followed by the given anomalies starting from the given longitude and increasing in a positive (+ east) longitude direction.

2. Profile type

3. New 5'x 5' Anomaly Block - This is the same as the original set, but with the elements replaced by 2:5x 2:5 means being set zero. Above each set of anomalies is given the latitude (degs and decimals), longitude (degs and decimals), and number of anomalies.

4. The 2:5x 2:5 anomaly blocks are next given. A strip along a constant latitude or longitude are given. If ITYPE = 9999,
(east-west strip), the anomalies are given along a constant latitude. If ITYPE = 999 (north-south strips), the anomalies are given along a constant longitude. The coordinates of the NW corner of the profile are given above the anomalies themselves along with the number of 2:5 anomalies in that strip.

5. Finally the answers are given in terms of computation point coordinates, elevations, upward continued gravity and normal gravity at that latitude and elevation.
6.4 Space Allocation

In a program of this type it is important to identify the needed space that must be set aside in the program, and to establish how the size or needed space may be computed. The following paragraphs describe these subjects for those variables that are size dependent.

In the main program:

\[
\text{ANOM}(A, 6) \\
A = (\text{length of 30' block in degrees}) \times 12 \\
\text{At the present time } A = 300 \text{ so that } (300/12 = 25^\circ).
\]

In the subroutine:

\[
\text{SUB343} \\
\text{DG}(B), \text{FL}(B), \text{F2}(B), \text{F3}(B), \text{F4}(B), \text{H}(B) \\
B \text{ is the maximum number of elevations at which the upward continued anomaly is to be computed.} \\
\text{At the present time, } B = 15.
\]

\[
\text{ANOM}(C,D) \\
C = (\text{length of 30' block in degrees}) \times 12 + 3 \\
D = (\text{number of 30' blocks (NBL)}) \times 6 \\
\text{At the present time, } C = 100 \text{ so that } (100-3)/12 = 8 \text{ degree band. } D = 48 \text{ so that the number of 30' blocks is 8 or } 4^\circ \text{ in latitude extent.}
\]

\[
\text{ANOM}(E,F) \\
E = \text{maximum number of } 2^1/2 \text{ mean anomalies in any strip + 3} \\
F = \text{number of } 2^1/2 \text{ strips to be used} \\
\text{At the present time, } E = 100, \text{ so } ((100-3) = 97 \text{ 2}^1/2 \text{ anomalies}) \\
or 48 \text{ 5'x 5' anomalies or essentially a 4' long band} \\
F = 12 \text{ 2}^1/2 \text{ strips or for a swath 30' wide.}
\]

\[
\text{NUMBER}(F), \text{ III}(F), \text{ JJJ}(F) \text{ should have the same space allocation given by the quantity } F.
\]

Additionally, if these variables are changed, so should several variables in the program: Thus: IVAR = F, IROWL = C, IROW2 = E. In addition, some clearing operations identifiable at the beginning of the program and at the beginning of the subroutine must be carried through all allocated space.
7. **Summary**

The purpose of this report has been to describe and to put into operation the necessary steps needed for the computation of the upward continued gravity anomaly, $\Delta g_H$, at some elevation $H$. The operational requirement of the above purpose is satisfied through the enclosed digital computer programs written in FORTRAN II which enables the computation of $\Delta g_H$ given the free-air anomalies on the surface of the geoid.

The equation that was used is essentially the Poisson Integral modified for use on a plane. The use of the plane is justified from the need for the anomalies only in the immediate vicinity of the computation point since the study was concerned only with heights that could be in the range found in airborne gravimetric work. The immediate vicinity is determined by going a distance, approximately 10 times the elevation at which the upward continuation is to take place, from the computation point.

Investigations were carried out to ascertain what the proper form of the equations to be used should take, and what size anomalies should be used for the upward continuation. It was found that accurate results could be obtained by using as integrated coefficient in the upward continuation equation for the squares within 5' of the squares in which the computation point fell, and in a non-integrated form for the other squares. It was also found that it is necessary to use not only 5'x 5' mean free-air anomalies, but 2 1/2 x 2 1/2 blocks must be used to maintain an accuracy of ± 1 mgal below 12 km in elevation. These anomalies are only those in the immediate (5') vicinity of the point. If the 2 1/2 means are used, the lowest elevation at which the upward continued anomaly may be computed to maintain an accuracy of one mgal is on the order of 5 km. These figures were determined by computing the upward continued anomaly under various conditions with respect to equation coefficients and anomaly fields. Under different circumstances it is possible that slightly different elevation conditions could be warranted. However, it is felt that the values given should be reasonable guidelines to be used when actually carrying out the uplifting.

The computer program that was designed to be used for the upward continuation of the gravity anomalies has the possibility of using both 2 1/2 and 5' anomaly blocks. Various control cards are used to identify each and insure their proper storage in the machine. In addition it is necessary to specify the computation as along a predominantly north-south
or east-west direction. This feature improves computation efficiency when profiles are being uplifted but has no specific need for single or clustered computations. The time for the computation depends on the size of the ground area involved, the number of points to be uplifted, and the number of elevations per point. In a test case using a $3^\circ \times 5^\circ$ area of five minute anomalies (2160) and $36 \ 21^\frac{1}{2}$ means, the time required was 0.4 minutes for 10 elevations. This figure would be on the order of 0.3 minutes for 1 elevation. Thus the manner in which the program is written makes the time of execution only very slightly dependent on the number of elevations being used.

This study has presented an operational program to be used in the upward continuation of gravity anomalies to heights on the order of 50 km. In addition some suggestions are made on the limitations and applicability of such a program.
REFERENCES


Appendix A

Source Statements for the Upward Continuation of Gravity Anomalies

The following program has been written in FORTRAN II operating under the monitor system at The Ohio State University Computer Center. In this installation the input tape is logical tape 5, which has been indicated by the variable ITAP in the program. The output tape is logical tape 6, which has been indicated by the variable KTAP in the program. In addition logical unit 9 is used internally in the program for intermediate storage.

The program is in single precision except for the subroutine which computes the normal gravity, this subroutine being in double precision. However, the output of this normal gravity is in single precision due to limitation of the OSU system. This is easily changed at installation where double precisions formats are available.
UPWARD CONTINUATION OF GRAVITY ANOMALIES
DIMENSION TEMP(6), ANOM(300, 6)
REWIND
ITAP=5
KTAP=6
READ INPUT TAPE ITAP=13, NBL
13 FORMAT(12)
DO511=1, NBL
DO19J=1, 300
DO19K=1, 6
ANOM(J,K)=0, 0
19 CONTINUE
READ INPUT TAPE ITAP=11, LATAD, LATAM, LOND, LONGM
11 FORMAT(12, 13, 14, 13)
JFX=300
JLX=1
23 READ INPUT TAPE ITAP=12, TEMP(K), K=1, 6, LATD, LATM, LOD, LOM
12 FORMAT(6F5.1, 2B4X, 2(I3, 12))
IF(LATD=99123, 38.25)
25 IF(LATAD=LATD) 29, 26, 29
26 IF(LATAM=LATM) 29, 32, 29
29 WRITE OUTPUT TAPE KTAP=30, TEMP(K), K=1, 6, LATAD, LATM, LOD, LOM
30 FORMAT(25H, 1, 1, 10X, 2(I3, 12))
GOTO23
32 J=(LOD-LONGD)*12+(LOM-LONGM)/5+1
34 ANOM(J,K) Temp(K)
IF(JFX-J) 30, 36, 39
35 JFX=J
36 IF(J-JLX) 23, 23, 37
37 JLX=J
GO TO 23
38 NUM=JLX-JFX+1
LNX=LONGM+3*JFX-3
ITEM=LNM/60
LND=LONGD+ITEM
LNX=LNM-ITEM*60
DO51K=1, 6
LTO=LATAD
LTM=LATAM-3*K+5
47 LTO=LX+6
LTD=LTO-1
49 WRITE TAPE 9, LTO, LTM, LND, LNM, NUM
WRITE TAPE 9, (ANOM(J,K), J=JFX, JLX)
50 CONTINUE
51 CONTINUE
END FILE9
REWIND
LAST=6*NBL
WRITE OUTPUT TAPE KTAP=31
531 FORMAT(1H1, 10X, 25H, ORIGINAL S BY 5 ANOMALIES)
DO532L=1, LAST
READ TAPE 9, LTD, LTM, LND, LNM, NUM
57.
READTAPE9(J*1,J=1,NUM)
WRITEOUTPUTTAPETAP53;LTD;LTD;LND;LNM;NUM;ANOM(J*1,J=1,NUM)
FORMAT(1/1JX*15(1JX*10F10*1))
REWIND9
CALLSUBS43(NBL;ITAP;KTAP)
REWIND9
CALLEIXIT
END
SUBROUTINESUBS43(NBL;ITAP;KTAP)
DIMENSIONTEMP(4);DG(15);F1(15);F2(15);F3(15)
DIMENSIONH(15)
DIMENSIONF4(15);ANOM1(100,48);ANOM2(100,12);NUMBER(12)
DIMENSIONIII(12);JJJ(12)
CON=111.19493
DEG=0.0416667
DLMD=0.083333
FLMD=0.0
PM=0.0
T=0.0
DIM=6*NBL
IVAR=12
IROW1=100
IROW2=100
REWIND9
DO5=1,100
DO5J=1,10
DO5:ANOM2(1,J)=0.0
DO1JK=1,1DIM
READTAPETLD;LTD;LND;LNM;NUM
NUMP3=NUM+3
READTAPETAP(ANOM1(J,K),J=4,NUMP3)
T1=LTD
T2=LTM
ANOM1(1,1)=T1+T2/60.0
T1=LND
T2=LNM
ANOM1(2,1)=T1+T2/60.0
ANOM1(3,1)=NUM
KK1=K
GGLAT=ANOM1(1,1)
GGLNG=ANOM1(2,1)
READINPUTTAPETAP;18;ITYPE
FORMAT(14)
WRITEOUTPUTTAPETAP;17;ITYPE
FORMAT(114.1,9X*19HTYPE OF PROFILE IS 16/10X*24HNEW 5 BY 5 ANOMALY 1LOCK)
LATITU=0.
IF(ITYPE=99921*19*20)
LATITU=1.
GOTO24.
IF(ITYPE=99921*24*21)
WRITEOUTPUTTAPETAP;22
FORMAT(10X*27HINCORRECT TYPE; JOB DELETED)
GOTO24
KK=1
I=9
J=1
NEW=0
IF (ANOM2 (1, J)) 29, 28, 29
28 IF (ANOM2 (1, J+1)) 29, 30, 29
30 NEW = 1
29 READ INPUT TAPE 1 TAP 31, (TEMP (K) K = 1, 4) LTD, LTM, LND, LNM
31 FORMAT (4F6.1, 34X, 2 (13, 12))
32 IF (LTD = 99) 32, 1470, 32
33 IF (J,J/2) 2-J) 320, 36, 320
33 IF (I, 2) T2 = 1) 34, 33, 34
33 IF (I = 3) 34, 34, 330
330 IF (I = 1) ROW 1331, 331, 34
331 IF (J) 34, 34, 332
332 IF (J = IVAR) 37, 37, 34
34 WRITE OUTPUT TAPE 35, (TEMP (K) K = 1, 4) LTD, LTM, LND, LNM
35 FORMAT (1UX*43HINCORRECT 2 1/2 BY 2 1/2 CARD - JOB DELETED/10X*4F6,
11, 1UX*41H)
36 GOTO 029
37 IF (NEW) 38, 50, 38
38 T1 = LTD
39 T2 = LTM
40 ANOM2 (1, J) = T1 + T2 / 60.0
41 T1 = LND
42 T2 = LNM
43 ANOM2 (2, J) = T1 + T2 / 60.0
44 IF (LATITU) 41, 41, 44
45 ANOM2 (1, J+1) = ANOM2 (1, J) + DDLG
46 ANOM2 (2, J+1) = ANOM2 (2, J) + DDLG
47 GOTO 046
48 ANOM2 (1, J+1) = ANOM2 (1, J)
49 ANOM2 (2, J+1) = ANOM2 (2, J) + DDLG
50 NUMBER (J) = 0
51 M = (ANOM2 (2, J) - GGLNG) / DLMD + 3.5 + 1.0
52 JJJ = JJJ / (GGLAT - ANOM2 (1, J)) / DLMD + 3.5 + 1.0
53 IF (M) 34, 49, 49
54 IF (J = JJJ) 34, 34, 34
55 NUMBER (J) = NUMBER (J) + 2
56 ANOM2 (1, J) = TEMP (1)
57 ANOM2 (1, J+1) = TEMP (4)
58 IF (LATITU = 94, 59, 39
59 ANOM2 (1, J+1) = TEMP (2)
60 ANOM2 (1, J+1) = TEMP (3)
61 NLONG = 111 (J) + (1 - 4) / 2
62 NLAT = JJJ (J)
63 GOTO 063
64 ANOM2 (1, J+1) = TEMP (3)
65 ANOM2 (1, J+1) = TEMP (2)
66 NLOCAL = 111 (J)
67 NLAT = JJJ (J) + (1 - 4) / 2
68 IF (NLOCAL = 100) 64, 64, 34
69 IF (NLAT = 101M) 65, 65, 34
70 ANOM1 (NLONG, NLAT) = 120.0
71 READ INPUT TAPE 1 TAP 31, (TEMP (K) K = 1, 4) LTD, LTM, LND, LNM
72 IF (LTD = 99) 67, 1470, 67
73 IF (LATITU = 90J) 680, 680, 1410
74 T1 = LTD
75 T2 = LTM
76 J = J + 5 + (ANOM2 (1, J) - (TI + T2 / 60.0)) / DDBE + 1.0
77 T1 = LND
78 T2 = LNM
I=3*5+(T1+T2/60.0-AOM2(2,J))/DDEG+1.0
GOTO1430

1410 T1=LND
T2=LN0
J=(T1+T2/60.0-AOM2(2,1))/DDEG+1.0
T1=LTD
T2=LTM
IF(3+ANOM2(1,J)-(T1+T2/60.9))/DDEG+1.0

1430 IF(J=KK)1440,1431,1431

1431 KK=J+1

1440 NEW=0
IF(ANOM2(1,J))1443,1441,1443
IF(ANOM2(1,J+1))1443,1442,1443
NEW=1
1443 IF(NEW)=32,32,1430

1450 I=4
GOTO32

1470 D01490 X=1, KK+2
ANOM2(3,K)=NUMBER(K)

1490 ANOM2(3,K+1)=NUMBER(K)
D0 1495 K=1, I+1

NUMTAP= ANOM1(3,K)
NUMTAP= NUMTAP+3

1495 WRITEOUTPUTAPEKTAP=1697* (ANOM1(J,K),J=1,NUMTAP)*

1497 FORMAT(16H1,19X,3F12.4/(10X,10F10.1))

WRITEOUTPUTAPEKTAP=1592

1502 FORMAT(16H1,9X,2F10.4/F5.2/(10X,10F10.3))

D01500 X=1, KK
NUMTAP=ANOM2(3,K)
NUMTAP=NUMTAP+3

WRITEOUTPUTAPEKTAP=1591* (ANOM1(J,K),J=1,NUMTAP)*

1505 CONTINUE

1501 CONTINUE
THE DATA IS ALL ENTERED. START CALCULATION.

68 WRITEOUTPUTAPEKTAP=6,69

69 FORMAT(16H1,10X,3HLATITUDE,15X,9H1ONGITUDE,12X,6HHEIGHT,9X,20HDELTA
2 G M (IN MGALS),8X,14HNORMAL GRAVITY,8X,215HDEG MIN SEC,8X)

70 READINPUTAPEKTAP=71*LTD,LTM,FLTS,FLS,LND,LMX,FLS+NHS,J(1),I=1,N

71 FORMAT(12,13F8.2, J=1,3,F6.2,13,F6.4, (5F8.4))
IF(LTD=0)71,2,73

72 T1=LTO
T2=LTX
PHI1=T1+(T2+FLS/60.0)/60.0
T1=LTD
T2=LMH
FLMD0=T1+(T2+LNS/60.0)/60.0
FLSFLSTAP=20,0.00,0,0,0,J,0
IF(FLSTAP)72,75,74

74 FLMD0=360.0-J,FLMD0

75 CONTINUE
D070 I=1,NHS

76 DG(I)=0.0
D074 K=1,3
PHI1=ANOM1(1,K)-DDEG
FLMD1=ANOM1(2,K)+DDEG

60.
NUM = ANOM1(3*K)
COSP = COSF(PHI1*U*174332922)
CONST = COSP*CON
Y = CON*(PHI1-PHI0)
YSQ = Y**2
NUMP3 = NUM+3
D093J = 4*NUMP3
X = CONST*(FLMD1-FLMD0)
XSQ = X**2
D0911 = 1*NHS
D1 = SQRTF(XSQ+YSQ+H(1)**2)
C = COSP*H(1)/(D1**3)
91 DG(1) = DG(1) + C*ANOM1(J,K)
FLMD0 = FLMD1+DLMD
93 CONTINUE
94 CONTINUE
D0961 = 1*NHS
96 DG(1) = B5*86328*DG(1)
C
C PROCEED TO CALCULATION FOR NEW ANOMS
C
C IF (ITYPE-99) 148, 142, 104
C
C THE CALCULATION IS FOR LONGITUDE CARDS
C
104 D0137K = 1*KK
PHI1 = ANOM2(1*K)
COSP = COSF(PHI1*U*174332922)
CONST = COSP*CON
NUM = ANOM2(3*K)
FLMD1 = ANOM2(2*K)
Y = CON*(PHI1-PHI0)
Y1 = CON*(PHI1-DDEG-PHI0)
YSQ = Y**2
Y1SQ = Y1*Y1
X = CONST*(FLMD1-FLMD0)
XSQ = X**2
D01211 = 1*NHS
HSG = H(1)**2
D1 = SQRTF(XSQ+Y1SQ+HSG)
D4 = SQRTF(XSQ+YSQ+HSG)
FI1 = ATAN2F(X*Y1+H(1)*D1)
121 F4(1) = ATAN2F(X*Y+H(1)*D4)
NUMP3 = NUM+3
D0136J = 4*NUMP3
X1 = CONST*(FLMD1+DDEG-FLMD0)
X1SQ = X1*X1
D01341 = 1*NHS
HSG = H(1)**2
D2 = SQRTF(X1SQ+Y1SQ+HSG)
D3 = SQRTF(X1SQ+YSQ+HSG)
F21(1) = ATAN2F(X1*Y1+H(1)*D2)
F31(1) = ATAN2F(X1*Y+H(1)*D3)
C = F1(1) - F2(1) + F3(1) - F4(1)
DG(1) = DG(1) + C*ANOM2(J,K)
F4(1) = F3(1)
134 FI111 = F2(1)
FLMD1 = FLMD1+DDEG
THE CALCULATION IS FOR LATITUDE CARDS

D0182K=1,KK
NUM =ANOM2(3,K)
FLM01=ANOM2(2,K)
TEMP1=CON*(FLM01-FLM00)
TEMP2=CON*(FLM01+OODEG-FLM00)
PHII=ANOM2(1,K)
CONST=CON
COSP=COSF(PHI,J*0.0174532925)
X=TEMP1*COSP
X=TEMP2*COSP
X5Q=X*X
X1SO=X1*X1
Y=CONST*(PHII-PHI0)
YSU=Y*Y
D0161I=1,NHS
HSQ=H(I)**2
D3=SQRFT(X1SO+Y5Q+HSQ)
D4=SQRFT(X5Q+YSO+HSQ)
F3(I)=ATAN2F(X1Y*H(I)*D3)
F4(I)=ATAN2F(X*Y*H(I)*D4)

D0181J=4,NUMP3
Y1=CONST*(PHII-DODEG-PHI0)
Y1SO=Y1*Y1
D0174I=1,NHS
HSQ=H(I)**2
D1=SQRFT(X5Q+Y1SO+HSQ)
D2=SQRFT(X1SO+Y1SO+HSQ)
F1(I)=ATAN2F(X*Y1*H(I)*D1)
F2(I)=ATAN2F(X1Y1*H(I)*D2)
C=F1(I)-F2(I)+F3(I)-F4(I)
DG(I)=DG(I)+C*ANOM2(J,K)
F4(I)=F1(I)

D184 F3(I)=F2(I)
PHII=PHII-DODEG
COSP=COSF(PHI,J*0.0174532925)
X=TEMP1*COSP
X1=TEMP2*COSP
X5Q=X*X
X1SO=X1*X1

CONTINUE

READY FOR OUTPUT

RLM00=0.0174532925*FLM00
RPHI0=0.0174532925*PHI0
IF(FLM00<100.0)169,189,1861
IF(FLM00=FLM00-360.0)
NEG=0
NEG=1
D SSQALF = (FKSQ-HSQ)/(2.0*PSQ)
D ALPHASF=SQRTF(SSQALF/(1.0-SSQALF))
D ALPHAT=ATANF(ALPHASF)
D COSALF=COSF(ALPHAT)
D SINALF=SQRTF(SSQALF)
D COTALF=COSALF/SINALF
D BETASF=SQRTF(PSQ)
D BETAT=ATANF(Z/(BETASF*COSALF))
D COSBET=COSF(BETAT)
D SINBET=SINF(BETAT)
D CSQBET=COSBET*COSBET
D Q=0.5*(ALPHAT-3.0*COTALF*(1.0-ALPHAT*COTALF))
D DP=(3.0*(1.0-ALPHAT*COTALF)/SSQALF)-1.0
D W=SQRTF(1.0-SSQALF*CSQBET)
D GAMMA = (FKM*SSQALF/(CSQ*))+(OMEGSQA*A*Q*Q*SSQALF)/(2.0*C*QQW)*(S)
D 2NBT*SINBET=1.0/3.0)
D GAMMA = (OMEGSQA*A*Q*SINBET*COSBET)/(C*QQW)
D GAMMT=SQRTF(GAMMA*GAMMA+GAMM*GAMM)
D SGAMA=GAMMA-(OMEGSQA*CSQBET)/(W*SINBET*COSBET))
D SGAMM=GAMMA+(OMEGSQA*SINBET*COSBET)/(W*SINBET))
D SGAMMT=SQRTF(SGAMA*SGAMA+SGAMM*SGAMM)
D RETURN
END
Appendix B

Sample Input Data.

Included in the appendix are selected listings of the program input data. The first page contains a sample of the first input to the program where the number of 30' blocks has been specified and the first 30' block with 5' mean anomaly cards has started to be given. The second page shows the input connected with the 2' 1/2 mean anomalies and the specific coordinates at which the upward continued anomaly is to be computed.
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99
Appendix C

Sample Output Data

Included in this appendix are selected listings from an actual execution of the program. The first page shows the form of the output of the original 5' mean anomalies. The second page shows the profile types and the modified 5' values (although in this case no zero elements appear). The third page gives the 2'1/2 mean anomalies while the fourth and last page in this appendix given the anomalies and other identified information computed by the program.
### Original 5 by 5 Animals

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